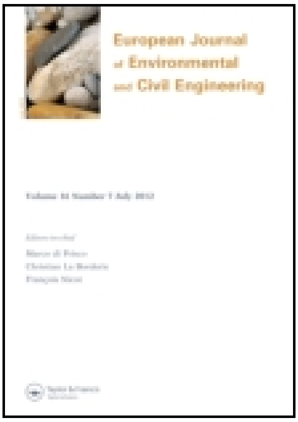

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Non-linear analysis of the behaviour of buried structures in random media

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In this study, a review of various research works dealing with the numerical modelling of soil–structure interaction problems is presented. Numerical models have been illustrated to show the performance of techniques used for resolving soil–pipe interaction problems by considering the non-linear and random aspects of soil. Non-linear analysis has been carried out by using an elasto-perfectly plastic model where soil geo-mechanical characteristics have been determined by means of a probabilistic approach "Monte Carlo method" to quantify the influence of the spatial variability of soil geo-mechanical characteristics on the longitudinal responses of a part of sewer. Numerical illustrations are based on the combination of the soil non-linearities and the probabilistic determination of its geo-mechanical properties which can allow us to describe the mechanical behaviour of buried structures in dispersed environments at the stage where the complexity of soil–structure interaction problem is relatively more pronounced.

Keywords: soil–structure interaction; buried sewers; material non-linearity; finite difference method; Mohr–Coulomb model; modulus of subgrade reaction; spatial variability

1. Introduction

Nowadays, pipelines are considered to be one of the best methods for underground transport of various goods and products of distinctive values and vital importance. Pipelines are one of the strategic and vital components of the urban infrastructure in different parts of the world. However, the interruption of water supply, waste water disposal or the interruption of supply in different power plants, refineries and petrochemical plants are due, in most cases, to structure damages of the underground pipeline networks. The origin of these underground pipeline breakages may be caused by differential settlements. These differential settlements are governed by a mechanism linked to the pipe coupling with the supporting soil, namely the soil–pipeline interaction problem which remains difficult to be studied because of its very complex geometries and geo-mechanical proprieties. This difficulty has led to numerous studies based on numerical approaches that lead to a quasi-realistic prediction of the mechanical behaviour of the soil–pipeline system under the influence of different solicitations like

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static, dynamic, cyclic or even thermal loads. Among these research studies, the Winkler (1867) model describes the soil modulus of subgrade reaction as a factor assuring the soil and pipe structural coupling in order to obtain models leading to very interesting results of structural analysis of soil–structure interaction. Among those authors who proposed to improve this model, Filonenko-Borodich (1940), Hetenyi (1950), Horvarth (1983, 1993), Kerr (1964, 1965), and Pasternak (1954) can be quoted. In the research works undertaken by Eisenberger and Yankelevsky (1985) and Huang and Shi (1998), finite element formulations – based on the elastic support beam theory – have been developed utilising the one-dimensional linear elements where stiffness matrices have been obtained with a very good precision (Eisenberger & Yankelevsky, 1985). More recently and in order to illustrate these models such as in the case of submarine catenaries from offshore oil platforms in the bottom of the oceans, Bridge, Laver, Clukey, and Evans (2004) and Bridge and Willis (2002) have proposed a model of catenary–soil mechanism under the influence of cyclic loading describing the penetration of the catenary and its contact with the ocean bottom. A similar and more realistic study has been carried out by Nakhaee and Zhang (2010) to describe the fatigue behaviour of catenaries at the time of their penetration into the soil. The geometric non-linearity has been introduced in the study of the catenary–soil interaction by Hosseini Kordkheili and Bahai (2008) through the finite-element model leading to the analysis of the catenary–soil system in the field of large deformations. Experimental studies taking into consideration three-dimensional answers of the submarine catenary have been carried out by Hodder and Byrne (2010) in order to calibrate the numerical models of the soil–catenary system. Recently, an attempt of pipeline finite element modelling was proposed by Joshi, Prashant, Deb, and Jain (2011) to utilise the three-dimensional beam elements maintained by non-linear elastic supports. The consideration of the soil variability along pipes has led to a model which describes, in a more realistic way, responses of the soil–pipeline system. Elachachi, Breysse, and Houy (2004) have proposed a model which takes into consideration the spatial variability of soil geo-mechanical characteristics along the pipeline through probabilistic methods. A particular attention has been paid to the influence of pipeline stiffness, the influence of soil and joint liaison over the responses of underground pipelines under static loading. In their research studies, Nedjar, Bensafi, Elachachi, Hamane, and Breysse (2002) and Nedjar, Hamane, Bensafi, Elachachi, and Breysse (2007) have resumed the same model which was studied in Elachachi et al. (2004), but this time under the influence of seismic loading. The dynamic behaviour and statistical analyses have been presented to illustrate the random variability of soil geo-mechanical characteristics and their importance in the design of underground pipelines. Soil non-linear aspects have been largely discussed by several authors in the field of soil–structures interaction problems, such as Cocchetti, di Prisco, Galli, and Nova (2009), Dickin (1994), Ilamparuthi and Dickin (2001), Scarpelli, Sakellariadi, and Furlani (2003) and Trautmann and O'Rourke (1983), those analysis are carried out by supposing that soil mechanical properties are constant along the pipe and soil variability has not been taken into account. In this paper, a new numerical strategy is presented in order to describe in a more rational way the mechanical behaviour of buried structures, especially, the case of interaction soil–pipe problem. This new model is based on the coupling of stochastic modelisation of soil geo-mechanical characteristics and soil non-linear behaviour which is based on the elastic-perfectly plastic load–displacement curves. The illustrations presented in this paper allow us to consider the random aspect of the supported soil in the non-linear analysis of soil–pipe interaction problems.

2. Formulation of numerical model

2.1. Finite difference formulation

The pipe element – in Figure 1 – represents the static numerical model of the soil–pipeline system, where k is the soil modulus of subgrade reaction determined in-situ or via lab tests. The pipe deflection is governed by classical equations below:

$$q(x) - ky = -\frac{\mathrm{d}V(x)}{\mathrm{d}x} \tag{1}$$

$$V(x) = -\frac{\mathrm{d}M(x)}{\mathrm{d}x} \tag{2}$$

$$M(x) = \mathrm{EI}\frac{\mathrm{d}^2y(x)}{\mathrm{d}x^2} \tag{3}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(\mathrm{EI}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right) = q(x) - ky \tag{4}$$

where $q(x) - ky$ is the resultant of the external load applied on the pipe and soil reaction force, $V(x)$ represents the internal shear effort, $M(x)$ is the bending moment and EI represents the rigidity upon the pipe flexion. The pipe is discretised into elements dx and for each node i of the pipe, the different orders of discreet model given by Equation (4) can be developed as follows:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y_{i+1} - y_{i-1}}{2(\mathrm{d}x)} = \frac{y_i - y_{i-1}}{\mathrm{d}x} \tag{5}$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{1}{\mathrm{d}x}\left[\frac{y_{i+1} - y_i}{\mathrm{d}x} - \frac{y_i - y_{i-1}}{\mathrm{d}x}\right] = \frac{y_{i+1} - 2y_i + y_{i-1}}{\mathrm{d}x^2} \tag{6}$$

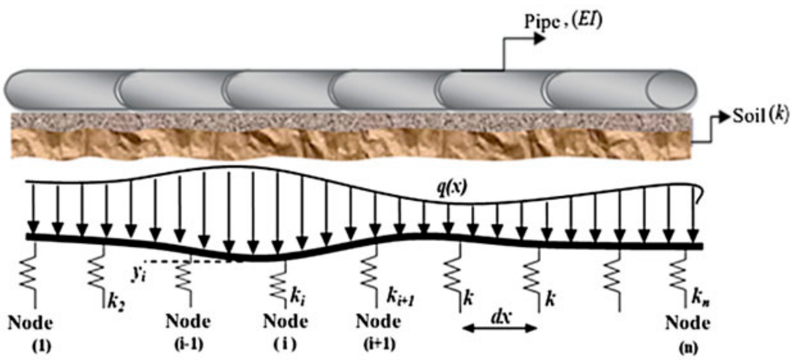

Figure 1. Modelisation of a pipe on an elastic soil.

And in the same way, we obtain

$$\frac{d^4y}{dx^4} = \frac{1}{dx}\left[\frac{y_{i+2} - 3y_{i+1} + 3y_i - y_{i-1}}{dx^3} - \frac{y_{i+1} - 3y_i + 3y_{i-1} - y_{i-2}}{dx^3}\right]$$
$$= \frac{y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2}}{dx^3} \quad (7)$$

Finally, the finite difference model of the pipe–soil system can be presented as follows:

$$\mathrm{EI}\left(\frac{y_{i-2} - 4y_{i-1} + 6y_i - 4y_{i+1} + y_{i+2}}{dx^4}\right) = -ky_i + q(x) \quad (8)$$

2.2. Non-linear formulation

In this work, the non-linear analysis concerns the behaviour law of the soil supporting the pipe which is governed by a simplified bi-linear model derived from a law of elastic perfectly plastic, based on the Mohr–Coulomb model – Figure 2 – where $P_{\max}$ and y_e are obtained through relations below in terms of major and minor principal local stresses σ_1 and σ_3 which are determined by laboratory tests. As shown in flowchart (Figure 3), $P_{\max}$ is computed via Equation (9) and soil modulus of subgrade reaction k_{soil} is generated for each node using VanMarcke theory (1983) of the local average for a random field. For each node of the soil–pipe model, the vertical displacements computed by finite difference processing (Equation (8)) are compared to elastic limit y_e (Equation (10)) and are used to update the soil modulus of subgrade reaction in order to carry out non-linear computation of soil–pipe system.

$$\sigma_1 - \sigma_3 = \frac{2c \cdot \cos\varphi - 2\sigma_3 \cdot \sin\varphi}{1 - \sin\varphi} \quad (9)$$

For the case where $\varphi = 0$,

$$y_e = \frac{2c}{k_{soil}} = \frac{\sigma_1 - \sigma_3}{k_{soil}} \quad (10)$$

c and φ are the cohesion of the soil and its friction angle, respectively. The average value of soil modulus of subgrade reaction k_{soil} is calculated from Vesic law (1961, 1963) in terms of Young modulus of soil E_s and pipe E_c, Poisson ratio of soil ϑ_{soil}, the inertia I_c of the pipeline and its external diameter D_{ext}.

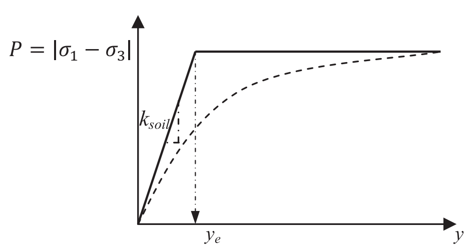

Figure 2. Soil non-linear behaviour law (EPP).

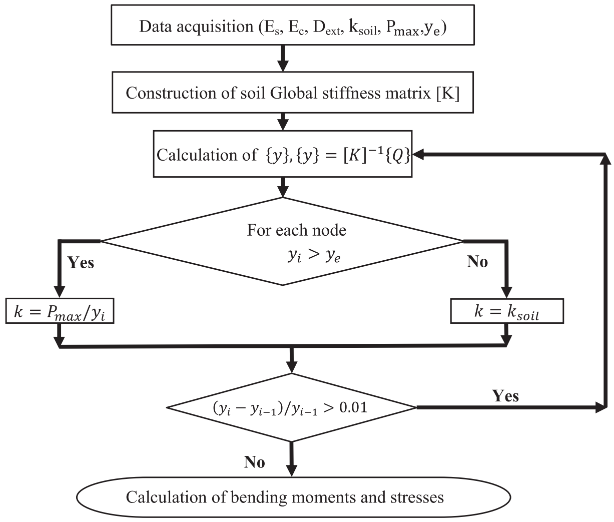

Figure 3. Process of non-linear calculation based on an elastic perfectly plastic behaviour law (EPP) for a 1% precision.

$$k_{\text{soil}} = \frac{0.65}{D_{\text{ext}}} \cdot \sqrt[12]{\frac{E_{\text{s}} D_{\text{ext}}^4}{E_{\text{c}} I_{\text{c}}}} \frac{E_{\text{s}}}{1 - \vartheta_{\text{soil}}^2} \tag{11}$$

The application of the soil's non-linearity is based on an iterative algorithm carried out under MATLAB interface which allows the update of the soil stiffness matrix $[K]$ of the subgrade reaction modulus k_{soil} with relation to the nodal displacements and deformation results associated with the soil yield deformation y_{e} as illustrated by the chart in Figure 3.

3. Random aspect of soil

As a result of the soil's natural process of formation and its aggregation, its spatial heterogeneity has been taken into consideration in this research work. This was carried out through probabilistic methods based on Monte Carlo approach in order to quantify the influence of spatial variability of soil stiffness by means of a non-exhaustive parametric study. The adopted approach is to combine the method of finite differences with the possibilities of stochastic modelling. These stochastic methods are essentially of two families, mainly the disturbance methods and Monte Carlo method based on three steps:

- Discretisation of random field.
- Analysis by finite difference method. (Deterministic calculation)
- Statistical analysis of structure responses after having carried out a consequent number of simulations for each achievement.

Among all the existing methods, the variation of the soil geo-mechanical characteristics can be properly described by the VanMarcke (1983) theory of local average. The random field of the soil subgrade modulus k_{soil} (x) is described by its average, its variance and the scale of fluctuation l_{c} which represents the distance beyond which the spatial correlation, between properties, is lost. In a zone (i) of a length D_i, the gaps of k_{soil} are respectively expressed by:

$$\text{Var}[k_{\text{sol}}(D_i)] = \sigma_k^2 \gamma(D_i) \tag{12}$$

And their local averages are respectively:

$$E_{\text{S}}[k_{\text{sol}}(D_i)] = m_{\kappa} \tag{13}$$

The average m_{k} is considered as constant for the entire field. The variance function y of the entire field of k_{soil} (x) is expressed as follows:

$$\gamma(D_i) = \frac{2}{D_i}\int_0^{D_i}\left(1 - \frac{x}{D_i}\right)\psi(x)\mathrm{d}x \tag{14}$$

$\gamma(D_i)$ represents the measurement of the variance reduction due to the average random process according to the length of the considered zone and is related to the correlation function $\psi(\tau)$, which varies between 0 and L_{c} and is given by:

$$\psi(\tau) = 1 - \frac{|\tau|}{L_{\text{c}}} \tag{15}$$

From Equations (14) and (15), the variance function can be obtained as follows:

$$\gamma(D_i) = \begin{cases} 1 - \frac{D_i}{3L_c} \quad \text{pour} \quad D_i \leqslant L_{\text{c}} \\ \frac{L_{\text{c}}}{D_i}\left(1 - \frac{L_{\text{c}}}{3D_i}\right) \quad \text{pour} \quad D_i \geqslant L_{\text{c}} \end{cases} \tag{16}$$

Therefore, it is easier to construct a random field for the whole system through co-variance matrices C_{ij} of soil reaction coefficients corresponding to the correlation between two zones of length D_i and D_j.

$$C_{ij} = \frac{\sigma_k^2}{2}[(t-1)^2\gamma[(t-1)D] - 2t^2\gamma(t.D) + (t+1)^2\gamma[(t+1)D]] \tag{17}$$

where $t = i - j$, i and j represent the zone numbers.

4. Numerical examples

In this section, numerical illustrations are presented for the case of a concrete pipeline of 1.0 m diameter over a total length of 62.0 m which is supposed continuous and does not contain any particular restraints at the intermediate connection joints. Its rigidity EI is about 869.45 MN m^2. The structure is simply supported at its ends, carried elastically over its length and subjected to the action of a uniformly distributed load ($Q = 7350$ N/m) resulting from the earth backfill weight over the pipeline and its own weight. The analysis was carried out based on a non-linear calculation following the Mohr–Coulomb model (elastic perfectly plastic) for both types of soil, very soft clay and stiff clay (Filliat, 1981; Lambe & Whitman, 1969; Winterkorn & Fang, 1975). In this section, it is very important to insist that non-linear responses of the concrete pipe has not been incorporated in the model and only material non-linearity of supported soil is considered in the simulations. Table 1 shows the main mechanical characteristics of materials used in the calculations.

The numerical resolution has been carried out by combining soil variability along the pipeline which has been incorporated through VanMarcke theory (1983) of the local average for a random field, with elasto-perfectly-plastic soil behaviour. The curves presented in Figures 4 and 5 show the pipeline responses in terms of the bending stresses and vertical deflections. These curves are done in the case of random variability of soil coefficient of subgrade reaction k_{soil} and in the case where the same parameter is supposed constant along the pipe (classical case). The statistical distribution of the pipe responses presents a strong dispersion characterised by an imposed variance of 7.76%. Pipe responses shown in Figures 4 and 5 form an envelope describing the ultimate responses in the nozzle, their practical usefulness allows to generate an optimal database of the stresses and deflections essential for the design of the pipeline sections through a probabilistic analysis based on the cumulative distribution functions of ultimate responses (Figure 6) given by each realisation.

Following the different simulations which take into consideration the reaction coefficient variability of the soil along the pipeline, the non-linear calculation is performed for each realisation obtained from the stochastic basis created based on the average and the variance values of soil modulus of subgrade reaction.

Table 1. Mechanical properties of the materials used in calculations.

	Young modulus (MPa)	Poisson ratio	Cohesion c (MPa)	Modulus of reaction k_{soil} (MN/m^3)
Concrete pipe	30,000	0.2	–	–
Stiff clay	12.5	0.35	0.05	6.502
Soft clay	6.0	0.4	0.0125	3.067

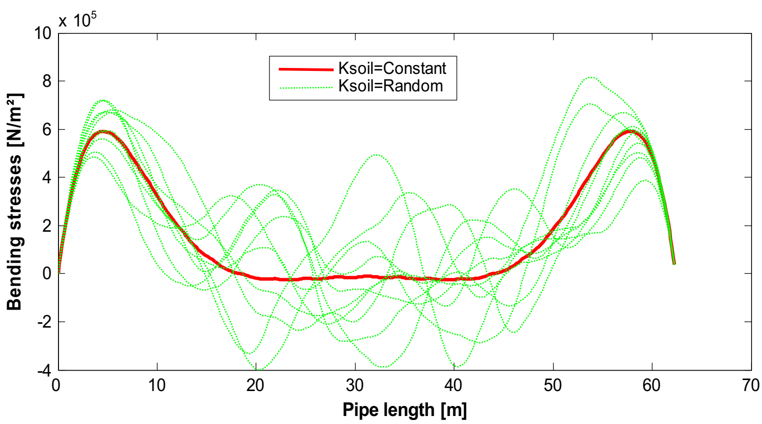

Figure 4. Bending stresses of the pipe for few hazards of soil coefficient of subgrade reaction k_{soil}.

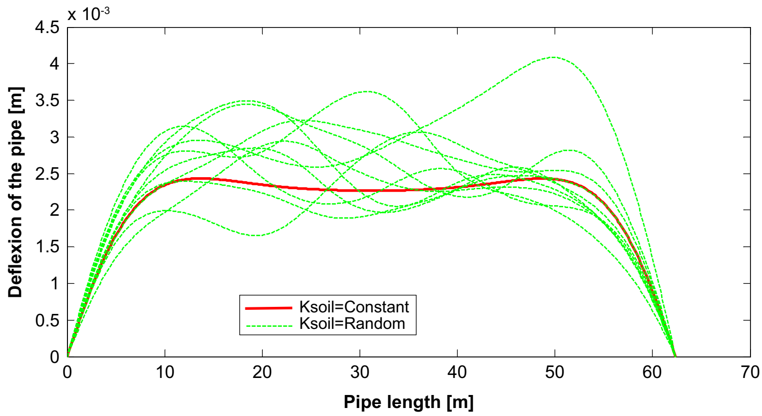

Figure 5. Deflection of the pipe for few hazards of soil coefficient of subgrade reaction k_{soil}.

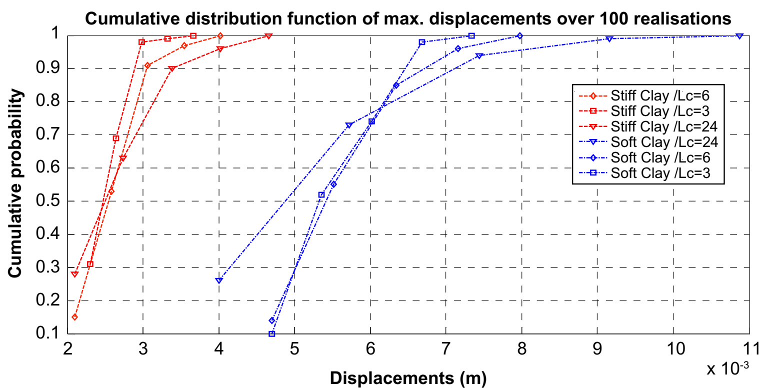

Figure 6. Cumulative distribution functions of maximum deflections for each type of soil in terms of the correlation lengths L_c.

The cumulative distribution function curves shown in Figure 6 allow us to quantify the probability of the maximal displacement values for all the hundred simulated realisation. Furthermore, this allows us to obtain the optimal design of the sewer sections in terms of the more representative internal stresses. The fluctuation of these internal stresses depends mainly on the repartition of the rigidity at the pipe–soil interface. The correlation lengths L_c and the simulation numbers have a significant influence on the calculation of the responses of the pipe and their determination should be obtained via a particular analysis in terms of the nature of the buried structure and its topology.

The present parametric study has been carried out for the case of stiff and soft clay soils and this, for several correlation lengths ($L_c = 3.0$, 6.0, 24.0 m).

For the three correlation lengths, the maximum displacement of 6.15×10^{-3}m (Figure 6) is obtained with a probability of 78% for the case, where the pipe is laying on soft clay soil. For the case of stiff clay soil, the vertical displacement of the pipe is about 2.55×10^{-3} m which is obtained with a probability of 51% and does not depend on the correlation length. Those displacements can be used to check the serviceability limit state of concrete pipe section.

Table 2. Comparative table between the deflections obtained by the current approach (probability of 70%) and deflections calculated in a linear elastic domain.

	Deflections obtained by the current approach (probability of 70%) (m)	Deflections obtained in linear elastic domain (k_{soil}: constant) (Timoshenko, 1940)
Stiff clay ($k_{soil} = 6.502$ MN/m^3)	2.91×10^{-3}	1.10×10^{-3}
Soft clay ($k_{soil} = 3.067$ MN/m^3)	5.93×10^{-3}	2.40×10^{-3}

For both cases of supported soils, the maximum vertical displacement of the pipe is always obtained when the correlation length is more significant ($L_c = 24$ m) and this, for the cases where cumulative probabilities are over 50% (Figure 6).

In order to compare deflections obtained by this current analysis with classical determination of vertical displacement of pipe sections which is carried out in linear elastic domain without considering the variability soil modulus of subgrade reaction along the pipe, we have noticed that the proposed approach gives always the worst case of maximal displacements which can be used to check the serviceability limit state of concrete pipe section (Table 2).

The non-linear analysis based on the proposed model allows us to have a probabilistic determination of failure possibilities in order to get a more suitable design of pipe sections in terms of safety and mechanical reliability.

5. Conclusion

In this work, a simple numerical model – allowing the modelling of the soil–pipe interaction – is presented based on a bibliographic synthesis of different research works carried out in the field of soil–structure interaction. Unlike classical methods (Timoshenko, 1940) used in numerical modelling of pipe–soil interaction problems which are carried out in linear elastic domain without considering soil variability along the pipe, the proposed approach is based on the hypothesis that the mechanical behaviour of underground pipelines can be described in a rational way by combining non-linearity aspect of soil and its variability. Cumulative probability of pipe deflections given for each realisation are plotted in Figure 6 for several correlation length in order to show the most representative pipe responses evaluated by the proposed model. As shown in Table 2, for both cases of supported soils, the computed pipe responses are relatively more significant and act in such a way to have a safe and optimal design of pipe sections. The curves indicate that soils of weak geo-mechanical characteristics penalise the pipe section design in terms of rigidity, in order to lead a good reliability of the pipe mechanical behaviour.

The correlation lengths have a significant effect on the probabilistic determination of pipe responses, especially in the case of pipes laid on soft clay soil where pipe response values are more significant.

The pipe mechanical behaviour is governed by many factors, namely the soil geo-mechanical characteristics and their variability as well as the correlation lengths. These factors can be used to quantify the structural hazards which may affect the buried pipe networks due to the excess of certain characteristics' values of displacements or stresses. These values – arising from a probabilistic analysis combined in a parametric study of the structure – will be intended to guide and define the design of the underground pipe networks towards a more optimum design for the pipe behaviour and its mechanical reliability.

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