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Contents:

(continued on outside back cover)

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Nonlinear Observer and Backstepping Control of Quadrotor Unmanned Aerial Vehicle

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Abstract – *This article presents the general concept of two nonlinear observers, such as high gain, sliding mode, adapted with nonlinear Backstepping control of quadrotor. The Positions is estimated by observers and set against reaction for control. The observatory technique using for estimated the non-measurable states has fine to reduce the number of sensors embedded in the x4-flying system. Several simulation tests in Simulink / Matlab are carried and takes into account an external disturbances (wind) to demonstrate the performance of the system controller-observer.*

***Keywords**: Nonlinear Observer, High Gain, Sliding Mode, Backstepping.*

Nomenclature

$B(O', x, y, z)$	Frame attached to G
$E(O, X, Y, Z)$	Inertial frame with *Ez* upwards
$\xi = [x, y, z]^T$	Position vector
$\alpha = [\varphi, \theta, \psi]^T$	Euler angles yaw, pitch and roll
R	Rotation matrix
G	Vehicle center of mass
m	Total mass of the vehicle
ω_i	Angular velocity of rotor i (i =1,4), *rad/s*
U_1	Collective force, *N*
$J = diag(Ix, Iy, Iz)$	Body inertia
F_i	Force developed by rotor, i
l	Distance from G to the rotor, i
τ_f	Moment according to *B* frame

I. Introduction

The intention of the researchers to humanize the machine, their will to reproduce the capacities human or animal of perception and action in the robotized systems and with the miniaturization of the sensors, the actuators and the technological maturity of embarked electronics; the advanced in the automatic and the artificial intelligence led to the quick evolution of the flying robots UAV [1]-[22]. These air vehicles caused a great interest thanks to their, maneuverability, capacity to carry out vertical takeoffs and landings and their large field of application as well military as civil, especially when the human intervention becomes difficult or dangerous.

A good control of a process passes in general by good information on this last. To ensure optimal control, it is also necessary to have knowledge complete or partial of the state of the system considered. However, in much of situations practical, for technical reasons or economic (construction, positioning and/or cost of the sensors), it is not always possible to reach all the variables of state of a system. Consequently, one is often brought to what is called rebuild the state of the system by using a «software sensor» or an observer.

The observer is a dynamic system based on the knowledge of the mathematical model describing the behavior of a system and using the entries and the measurements acquired on this one in order to rebuild the variables of state.

The use of an observer can be planned to answer three categories of objectives to knowing, the monitoring, the detection of failures and control (Fig. 1).

The rebuilding of the state of a dubious system is also a traditional problem of the automatic. Luenberger [15] studied a reconstruction of state, to which its name was allotted. The observer of Luenberger is not always sufficient, because the error in estimation generated by this observer for a dubious system or at unknown entries does not converge forcing towards the zero value.

To cure this problem, we can use the observers of the singular systems [16] or the observers at unknown entries [17], the problem of design of observers with action proportional and integral.

An observer-controller in [19], the observer is with sliding mode of a higher order, it with the absolute position and the angle of the lace, it rebuilds speeds of the helicopter and estimating the disturbances. As in [20], the observer is a differentiator with slipping mode of order 2 which considers the orders virtual of the Backstepping.

In this paper, the model form presented [2] that established by Lozano [6] by the method of Euler-Lagrange and that of Bouabdallah [4].

Then, the Backstepping controller and motion planning are combined to stabilize the helicopter by using the point to point steering stabilization. Modeling is briefly described Section II.

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Section III describes Backstepping controllers. Section IV, synthesis of non linier observer is presented, In Section V, simulation results are presented followed by Robustness consideration in Section V. Finally, some conclusions and future work are given in the last section.

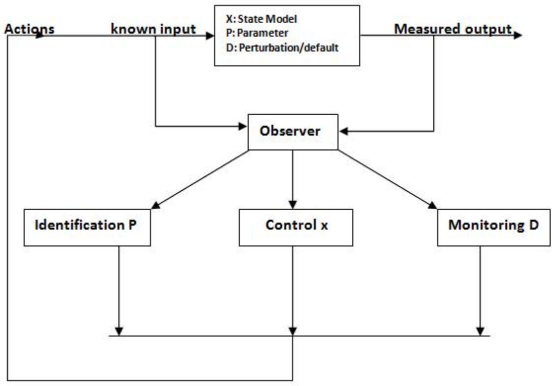

Fig. 1. Objectives of observer

II. Dynamic Modeling of Quadrotor

The mini quadrotor is four rotors helicopter, Fig. 2, each rotor consists of an electrical DC motor, a drive gear and propeller. The two pairs of propellers (1, 3) and (2, 4) turn in opposite directions. Forward motion is accomplished by increasing the speed of the rear rotor while simultaneously reducing the forward rotor by the same amount.

Aft, left and right motion work in the same way. Yaw command is accomplished by accelerating the two clockwise turning rotors while decelerating the counter-clockwise turning rotors. This helicopter is one of the most complex flying systems that exist. This is due partly to the number of physical effects (Aerodynamic effects, gravity, gyroscopic, friction and inertial counter torques) acting on the system [2]. We consider a local reference airframe $B(O', x, y, z)$ attached to the center of mass G of the vehicle. The center of the mass is located at the intersection of the two rigid bars. The inertial frame is denoted by $E(O, X, Y, Z)$ such that the vertical direction E_z is upwards. The absolute position of center of mass of quadrotor is described by $\xi = [x, y, z]^T$, and its attitude by the three Euler's angles $\alpha = [\varphi, \theta, \psi]^T$, these three angles are respectively pitch angle, roll angle and yaw angle.

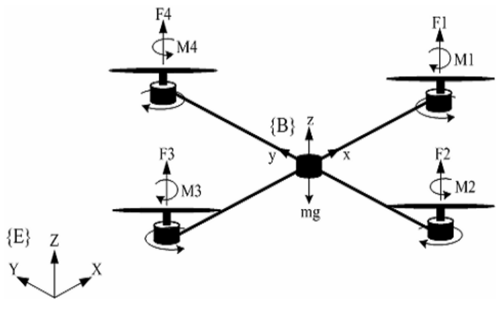

Fig. 2. Quadrotor configuration model

The transformation rotation between the earth reference frame and the body reference frame is given by the following matrix:

$$R = \begin{pmatrix} C_\theta C_\psi & C_\psi S_\theta S_\phi - C_\phi S_\psi & C_\phi C_\psi S_\theta + S_\phi S_\psi \\ C_\theta S_\psi & S_\theta S_\phi S_\psi + C_\phi C_\psi & C_\phi S_\theta S_\psi - C_\psi S_\phi \\ S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{pmatrix} \quad (1)$$

with $S(.)$ and $C(.)$ represent $sin(.)$ and $cos(.)$ respectively.

II.1. Translation Motion

Under these assumptions, it is possible to describe the fuselage dynamics as that of a rigid body in space to which come to be added the aerodynamic forces caused by the rotation of the rotors. Using the formalism of Newton-Euler, the dynamic equations are written in the following form:

$$\begin{cases} \dot{R} = RS(\Omega) \\ F_f + F_{dt} + F_G = m\ddot{\xi} \\ \tau_f - \tau_a - \tau_g = J\dot{\Omega} + \Omega \wedge J\Omega \end{cases} \quad (2)$$

where m is the mass of the structure and $J = diag(Ix, Iy, Iz) \in R3 \times 3$ is a symmetric positive definite constant inertia matrix of the quadrotor with respect to B, F_f, F_{dt} and F_G are respectively the forces generated by the propeller system, the drag force and the gravity force, such as:

$$F_{dt} = K_{dt}\dot{\xi} \quad (3)$$

where $K_{dt} = diag(K_{dtx}, K_{dty}, K_{dtz})$ are the translation drag coefficients:

$$F_G = mG \quad (4)$$

with $G = [0, 0, g]^T$ is gravity vector.

The forces generated by the propeller system of the quadrotor are given by the following equations [2]:

$$F_f = \begin{bmatrix} C_\phi C_\psi S_\theta + S_\phi S_\psi \\ C_\phi S_\theta S_\psi - C_\psi S_\phi \\ C_\theta C_\phi \end{bmatrix} \sum_{i=1}^{4} F_i \quad (5)$$

F_i is the lift force generated by the rotor i and it's proportional to the square of the angular speed rotation.

Using the dynamic Eq. (2) of translation becomes:

$$m\ddot{\xi} = F_f - K_{dt}\dot{\xi} - mG \quad (6)$$

II.2. Rotational Motion

Using the Newton's law about the rotation motion, the sum of moments is given as follow (2), the ∧ denotes the product vector, and **Ω** is the angular speed expressed in body fixed frame [3]:

$$\Omega = M\dot{\alpha} \quad (7)$$

with:

$$M = \begin{pmatrix} 1 & 0 & -S_\theta \\ 0 & C_\phi & C_\theta S_\phi \\ 0 & -S_\phi & C_\Phi C_\theta \end{pmatrix}$$

τ_f the moment developed by the quadrotor according to the body fixed frame is given by:

$$\tau_f = \begin{bmatrix} l(F_3 - F_1) \\ l(F_4 - F_2) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix} \tag{8}$$

with l the distance between the quadrotor center of mass and the rotation axe of propeller and M_i the quadrotor moment developed about z axis, τ_a is the aerodynamic friction torque:

$$\tau_a = K_{af}\Omega \tag{9}$$

and $K_{af} = diag(K_{afx}, K_{afy}, K_{afz})$ is the aerodynamic friction coefficients. τ_g is the gyroscopic torque, given by:

$$\tau_g = \sum_{i=1}^{4} \Omega \wedge J_r \begin{bmatrix} 0 \\ 0 \\ \omega_i \end{bmatrix} \tag{10}$$

with ω_i is angular speed of rotor i, and J_r is the rotor inertia. Consequently the complete dynamic model which governs the quadrotor is as follows:

$$\begin{cases} \ddot{\phi} = \frac{1}{J_x}\{(J_y - J_z)\dot{\theta}\dot{\psi} + J_r\dot{\theta}\bar{\Omega} + dU_2\} \\ \ddot{\theta} = \frac{1}{J_y}\{(J_z - J_x)\dot{\phi}\dot{\psi} - J_r\dot{\phi}\bar{\Omega} + dU_3\} \\ \ddot{\psi} = \frac{1}{J_z}\{(J_x - J_y)\dot{\phi}\dot{\theta} + U_4\} \\ \ddot{x} = \frac{1}{m}\{(C_\phi S_\theta C_\psi + S_\phi S_\psi)U_1\} \\ \ddot{y} = \frac{1}{m}\{(C_\phi S_\theta C_\psi - S_\phi S_\psi)U_1\} \\ \ddot{z} = \frac{1}{m}\{(C_\phi S_\theta U_1\} - g \end{cases} \tag{11}$$

The vectors U_1, U_2, U_3 and U_4 are the control of the system to the engine airframe including forces generated by the motors and drag terms.

III. Control of the Quadrotor

The choice of this method is considering because the major advantages it presents [2]:

- It ensures Lyapunov stability.
- It ensures the robustness and all properties of the desired dynamics.
- It ensures the handling of all system nonlinearities.

The model (11) developed in the first part of this paper can be rewritten in the state-space form: $\dot{x} = f(x) + g(x,u)$ and $x = [x_1, \dots, x_{12}]^T$, is the state vector of the system such as:

$$x = [\,\phi\ \dot{\phi}\ \theta\ \dot{\theta}\ \psi\ \dot{\psi}\ x\ \dot{x}\ y\ \dot{y}\ z\ \dot{z}\,]^T \tag{12}$$

From (11) and (12) we obtain the following state representation:

$$f = \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = a_1 x_4 x_6 + a_2 x_4 \bar{\Omega} + b_1 U_2 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = a_3 x_2 x_6 + a_4 x_2 \bar{\Omega} + b_2 U_3 \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = a_5 x_2 x_4 + b_3 U_4 \\ \dot{x}_7 = x_8 \\ \dot{x}_8 = \frac{U_1}{m} U_x \\ \dot{x}_9 = x_{10} \\ \dot{x}_{10} = \frac{U_1}{m} U_y \\ \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = \frac{c\,x_1\,c\,x_3}{m} U_1 - g \end{cases} \tag{13}$$

with:

$$\begin{cases} a_1 = \left(\frac{I_y - I_z}{I_x}\right), a_2 = \left(\frac{I_r}{I_x}\right) \\ a_3 = \left(\frac{I_z - I_x}{I_y}\right), a_4 = \left(-\frac{I_r}{I_y}\right) \\ a_5 = \left(\frac{I_x - I_y}{I_z}\right) \\ b_1 = \left(\frac{d}{I_x}\right), b_2 = \left(\frac{d}{I_y}\right), b_3 = \left(\frac{1}{I_z}\right) \end{cases} \tag{14}$$

and:

$$\begin{cases} U_x = (Cx_1 Sx_3 Cx_5 + Sx_1 Sx_5) \\ U_y = (Cx_1 Sx_3 Cx_5 - Sx_1 Sx_5) \end{cases} \tag{15}$$

We note that the system is in a form triangular waterfall can be controlled by Backstepping control.

III.1. Backstepping Controller

The Backstepping Kanellakopoulos was developed by [05], and inspired by the work of Feurre & Morse [7], Sussmann and Kokotovic & [11]. The basic idea is to let some system states act as virtual inputs.

The Backstepping uses a form of system integrators chain after a coordinate transformation for a triangular system based on the direct method of Lyapunov. The method consists of fragmented system into a set of subsystem nested descending order.

From the, it is possible to design systematically and recursively controllers and corresponding Lyapunov functions [9] [10] [12].

The algorithm Backstepping is described step by step in the following. We consider the tracking error:

$$z_i = \begin{cases} x_{id} - \hat{x}_i & /i \in \{1,3,5,11\} \\ \hat{x}_i - \dot{x}_{(i-1)d} - \alpha_{(i-1)} z_{(i-1)} & /i \in \{2,4,6,12\} \end{cases} \quad (16)$$

Using the all Lyapunov functions as:

$$V_i = \begin{cases} \frac{1}{2} z_i^2 & /i \in \{1,3,5,11\} \\ \frac{1}{2}(z_i^2 + V_{i-1}^2) & /i \in \{2,4,6,12\} \end{cases} \quad (17)$$

Application of algorithm:

For $i = 1$:

$$\begin{cases} z_1 = x_{1d} - \hat{x}_1 \\ V_1 = \frac{1}{2} z_1^2 \end{cases}$$

and:

$$\dot{V}_1 = z_1 \dot{z}_1 = z_1(\dot{x}_{1d} - \hat{x}_2) \quad (18)$$

Using the Lyapunov functions and especially $\dot{V}_i < 0$, the stabilization of z_1 can be obtained by introducing a virtual control input x_2 such that:

$$\hat{x}_2 = \dot{x}_{1d} + \alpha_1 z_1 \quad \text{with } \alpha_1 > 0$$

The Eq. (18) is then: $\dot{V}_1(z_1) = -\alpha_1 z_1^2$. We consider a variable change by making:

$$z_2 = \hat{x}_2 - \dot{x}_{1d} - \alpha_1 z_1^2$$

For $i = 2$:

$$\begin{cases} z_2 = \hat{x}_2 - \dot{x}_{1d} - \alpha_1 z_1^2 \\ V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 \end{cases}$$

and:

$$\dot{V}_1 = z_1 \dot{z}_1 + z_2 \dot{z}_2$$

Finally:

$$\dot{z}_2 = a_1 \hat{x}_4 \hat{x}_6 + a_2 \hat{x}_4 \bar{\Omega} + b_1 U_2 - \ddot{x}_{1d} + \alpha_1 \dot{z}_1 \quad (19)$$

We achieve the control U_2 that give (19) equal to $\alpha_2 z_2$, with α_2 is positive constant, the final control is:

$$U_2 = \frac{1}{b_1}\{-a_1 \hat{x}_4 x_6 - a_2 \hat{x}_4 \bar{\Omega} + \ddot{\psi}_d + \\ + \alpha_1(\dot{\psi}_d - x_2) - \alpha_2 z_2 + z_1\} \quad (20)$$

The term $\alpha_2 z_2$ is added in order to stabilize z_1. The same steps are taken again in order to extract U_1, U_3, U_4 (eq. (21)):

$$\begin{cases} U_1 = \frac{m}{(Cx_1 Cx_3)}[g + \ddot{z}_d + \alpha_7(\dot{z}_d - \hat{x}_{12}) - \alpha_8 z_8 + z_7] \\ U_3 = \frac{1}{b_2}\left[\begin{matrix} -a_3 \hat{x}_2 \hat{x}_6 - a_4 \hat{x}_2 \bar{\Omega} + \ddot{\theta}_d + \alpha_3(\dot{\theta}_d - \hat{x}_4) \\ -\alpha_4 z_4 + z_3 \end{matrix}\right] \\ U_4 = \frac{1}{b_3}[-a_5 \hat{x}_2 \hat{x}_4 + \ddot{\psi}_d + \alpha_5(\dot{\psi}_d - \hat{x}_6) - \alpha_6 z_6 + z_5] \end{cases}$$

Or them α_i (with $\in [1\ldots 8]$) are positive constant.

IV. Observer Design

IV.1. A high Gain Observer

The high gain observer [13] [22] is used in the case of uniformly observable systems [18]. This type of observer is interesting because it can be applied to a broad class of systems which includes the drone system studied.

The dynamic model of quadrotor on state space given in (13), and denote $\hat{x}$ the estimate of the state vector (12).

The observer design of the state variables can be represented in the following state space form:

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + \Lambda_1 \\ \dot{\hat{x}}_2 = a_1 \hat{x}_4 \hat{x}_6 + a_2 \hat{x}_4 \bar{\Omega} + b_1 U_2 + \Lambda_2 \\ \dot{\hat{x}}_3 = \hat{x}_4 + \Lambda_3 \\ \dot{\hat{x}}_4 = a_3 \hat{x}_2 \hat{x}_6 + a_4 \hat{x}_2 \bar{\Omega} + b_2 U_3 + \Lambda_4 \\ \dot{\hat{x}}_5 = \hat{x}_6 + \Lambda_5 \\ \dot{\hat{x}}_6 = a_5 \hat{x}_2 \hat{x}_4 + b_3 U_4 + \Lambda_6 \\ \dot{\hat{x}}_7 = \hat{x}_8 + \Lambda_7 \\ \dot{\hat{x}}_8 = \frac{U_1}{m} U_{\hat{x}} + \Lambda_8 \\ \dot{\hat{x}}_9 = \hat{x}_{10} + \Lambda_9 \\ \dot{\hat{x}}_{10} = \frac{U_1}{m} U_{\hat{y}} + \Lambda_{10} \\ \dot{\hat{x}}_{11} = \hat{x}_{12} + \Lambda_{11} \\ \dot{\hat{x}}_{12} = \frac{C\hat{x}_1 C\hat{x}_3}{m} U_1 - g + \Lambda_{12} \end{cases} \quad (22)$$

The observer error dynamics are given by:

$$\begin{cases} \dot{e}_1 = e_2 - \Lambda_1 \\ \dot{e}_2 = a_1 \Delta_{x_4 x_6} + a_2 e_4 \bar{\Omega} - \Lambda_2 \\ \dot{e}_3 = e_4 - \Lambda_3 \\ \dot{e}_4 = a_3 \Delta_{x_2 x_6} + a_4 e_2 \bar{\Omega} - \Lambda_4 \\ \dot{e}_5 = e_6 - \Lambda_5 \\ \dot{e}_6 = a_5 \Delta_{x_2 x_4} - \Lambda_6 \\ \dot{e}_7 = e_8 - \Lambda_7 \\ \dot{e}_8 = \frac{U_1}{m}(U_x - U_{\hat{x}}) - \Lambda_8 \\ \dot{e}_9 = e_{10} - \Lambda_9 \\ \dot{e}_{10} = \frac{U_1}{m}(U_y - U_{\hat{y}}) - \Lambda_{10} \\ \dot{e}_{11} = e_{12} - \Lambda_{11} \\ \dot{e}_{12} = \frac{U_1}{m}(U_z - U_{\hat{z}}) - \Lambda_{12} \end{cases} \quad (23)$$

with:

$$\begin{cases} \Delta_{x_4 x_6} = x_4 x_6 - \hat{x}_4 \hat{x}_6 \\ \Delta_{x_2 x_6} = x_2 x_6 - \hat{x}_2 \hat{x}_6 \\ \Delta_{x_4 x_6} = x_4 x_2 - \hat{x}_4 \hat{x}_2 \end{cases} \quad (24a)$$

and:

$$U_z = Cx_1Cx_3\,, U_{\hat{z}} = C\hat{x}_1C\hat{x}_3 \quad (24b)$$

with:

$$\begin{cases} a_1\Delta_{x_4x_6} + a_2e_4\bar{\Omega} = g_1 \\ a_3\Delta_{x_2x_6} + a_4e_2\bar{\Omega} = g_2 \\ a_5\Delta_{x_2x_4} = g_3 \end{cases} \quad (25)$$

The corrector gains it's calculated for the estimation errors dynamics is stable so, it was noted that the observer gains are functions of measurement errors such as:

$$\Lambda_i = l(e_1, e_3, e_5, e_7, e_9, e_{11}) \quad (26)$$

We take the first two equations of dynamic errors and explain how to choose the observer gains:

$$\begin{cases} \dot{e}_1 = e_2 - \Lambda_1 \\ \dot{e}_2 = g_1 - \Lambda_2 \end{cases} \quad (27)$$

with:

$$\begin{cases} \Lambda_1 = K_1e_1 \\ \Lambda_2 = K_2e_1 \end{cases} \text{ with: } (K_1, K_2)\mathcal{R}^{+2} \quad (28)$$

Gains K_1, K_2 are chosen such that: $K_1 = \frac{a_1}{\varepsilon}, K_2 = \frac{a_2}{\varepsilon^2}$.

First one chooses the gains to be exponential convergence of the linear part of the error. Second, we consider the nonlinear part as a perturbation and thus try to annul: if we choose $\varepsilon \ll 1$, and we do a change of variable $\Upsilon_1 = \frac{e_1}{\varepsilon}, \Upsilon_2 = e_2$.

We note that although the decrease in the parameter ε reduces the error of observation, and there is a convergence in minimum time.

The same steps are followed to extract others corrector gains.

IV.2. Sliding Mode Observer

In order to increase the robustness against modeling errors and uncertainty, observers based on the theory of variable structure systems are proposed [14] [21].

They are generally used for uncertain nonlinear dynamical systems. We present syntheses of observer [1][08] applied for nonlinear systems described by (13).

We define a state observer whose structure is:

$$\dot{\hat{x}} = \hat{f}(\hat{x}, y, u) + \Lambda I_s \text{ with } \hat{x} \in \mathcal{R}^n \quad (29)$$

and $\hat{f}$ of the model f, Λ is the gain matrix *(nxr)* determining and I_s is a vector ($rx1$):

$$I_s = [sign(s_1), sign(s_2) \dots sign(s_i)]^T$$

with $s_i = (y_i - \hat{y}_i)$.

The surface dimension r given by $S = 0$, is attractive if: $\dot{S}_iS_i < 0, i \in \{1, \dots, r\}$, Lyapunov condition verify,

This condition defines the area of the sliding mode.

The second stapes, the gain matrix patches is to satisfy the following invariance condition:

$$\dot{s} = 0 \text{ and } s = 0$$

IV.3. Application

Consider the model on state space (13) of the quadrotor, and the observer design of the state variables represented in (22) state space, and the observer error dynamics are given in (23), we can calculate the gain matrix by nonlinear sliding mode observer, we take the two first Eqs. (27) (28) of dynamic errors and explain how to choose the observer gains:

$$\begin{cases} \dot{e}_1 = e_2 - \Lambda_1 \\ \dot{e}_2 = g_1 - \Lambda_2 \end{cases} \quad (30)$$

with:

$$\begin{cases} \Lambda_1 = \lambda_1 sige(e_1) \\ \Lambda_2 = \lambda_2 sige(e_1) \end{cases} \quad (31)$$

Errors should converge to the equilibrium values in two steps:

Step 1: We consider only the first observation error:

$$\dot{e}_1 = e_2 - \lambda_1 sige(e_1) \quad (32)$$

Slippage observation errors on the sliding surface is guaranteed by λ_1. Let the Lyapunov function: $V_1 = \frac{e_1^2}{2}$, and their derivative $\dot{V}_1 = e_1(e_2 - \lambda_1 sige(e_1))$, with a choice of $\lambda_1 > |e_2|$ for $t < t_1$ observation error converges to zero after a finite time t_1.

Step 2: In this step λ_2 imposes the dynamic observation errors on the sliding surface. We set: $V_2 = \frac{e_1^2}{2} + \frac{e_2^2}{2}$, and drive $\dot{V}_2 = e_2(g_1 - \lambda_2 sige(e_1)) = e_2(g_1 - \lambda_2\frac{e_2}{\lambda_1})$:, choosing $\frac{\lambda_2}{\lambda_1} > |g_1|$. So after t_1, the sliding surface is reached and the error e_2 converges to zero in a finite time $t_2 < t_1$. The same steps are followed to extract others corrector gains, so:

$$\begin{cases} \Lambda_3 = \lambda_3 sige(e_3) \\ \Lambda_4 = \lambda_4 sige(e_3) \\ \Lambda_5 = \lambda_5 sige(e_5) \\ \Lambda_6 = \lambda_6 sige(e_5) \\ \Lambda_7 = \lambda_7 sige(e_7) \\ \Lambda_8 = \lambda_8 sige(e_7) \\ \Lambda_9 = \lambda_9 sige(e_9) \\ \Lambda_{10} = \lambda_{10} sige(e_9) \\ \Lambda_{11} = \lambda_{11} sige(e_{11}) \\ \Lambda_{12} = \lambda_{12} sige(e_{11}) \end{cases} \quad (33)$$

V. Simulations Results

The simulation results are obtained based on the following real parameters [9], an application has been run with wind disturbance (F_z) along the z direction, the strength is set to 6N, occurring at t=10s.

V.1. A High Gain Observer

Fig. 3, illustrates the controlled states using backstepping controller where (U_1, Ux, Uy), denote the command signals for z, x and y directions respectively.

Fig. 4, shows the perfect convergence from tracking trajectories, and the good estimated for the high gain observer.

Fig. 5, show the Zoom of the acceptable error in direction z response, this error present because of the introduce of the wind disturbance, The annihilation of be possible her in addition the integrator effect, but the convergence of observers states is perfectly.

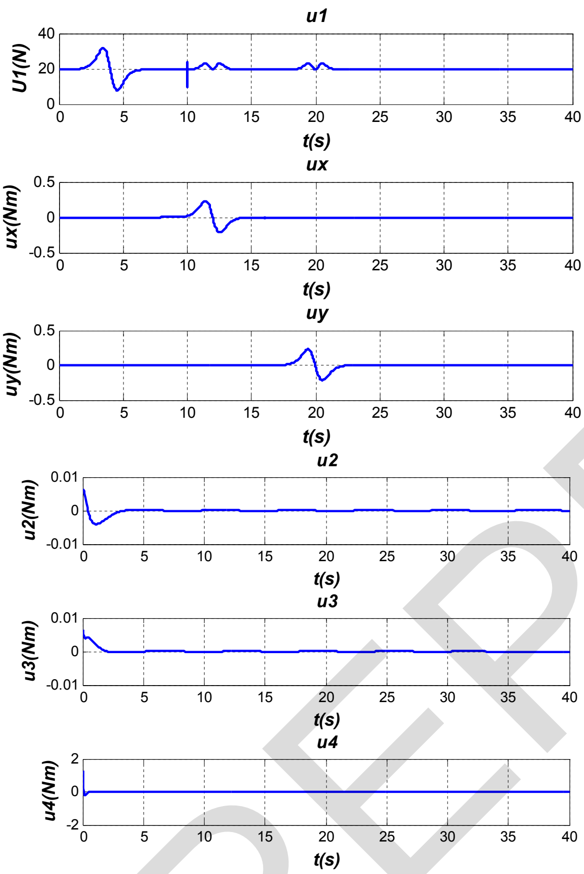

Fig. 3. Backstepping control signals with wind disturbance (6N)

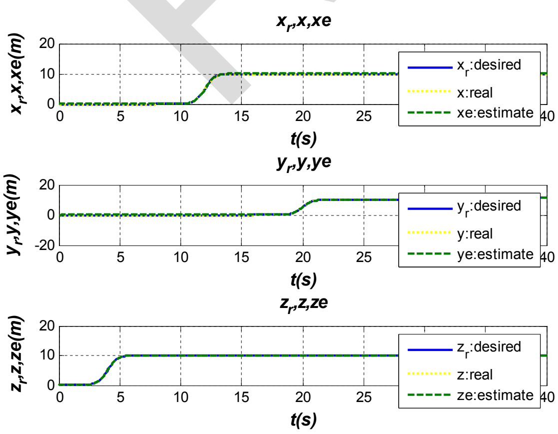

Fig. 4. Tracking simulation results with wind disturbance (6N)

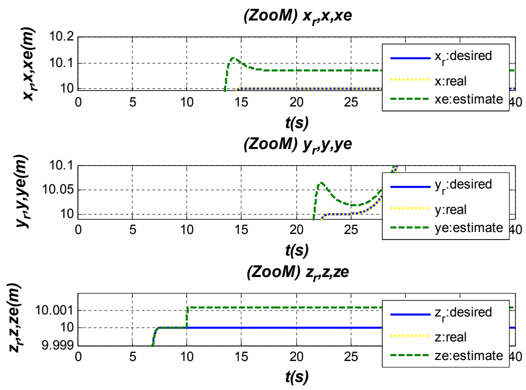

Fig. 5. Tracking simulation results Zoom with wind disturbance (6N)

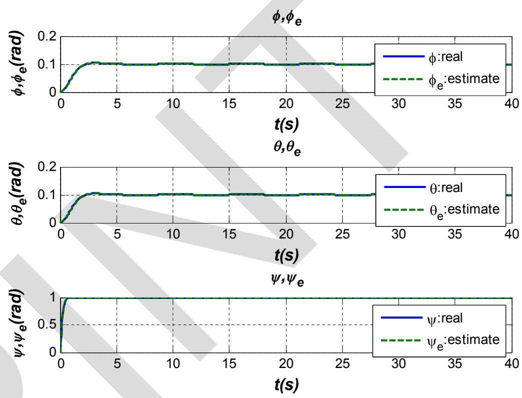

Fig. 6. The pitch (theta) roll (phi) and yaw (psi) angles

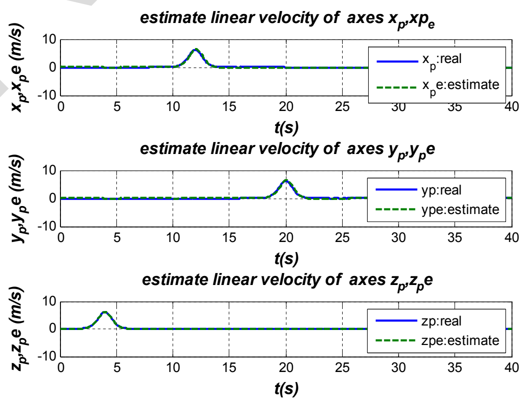

Fig. 7. Linear velocity results

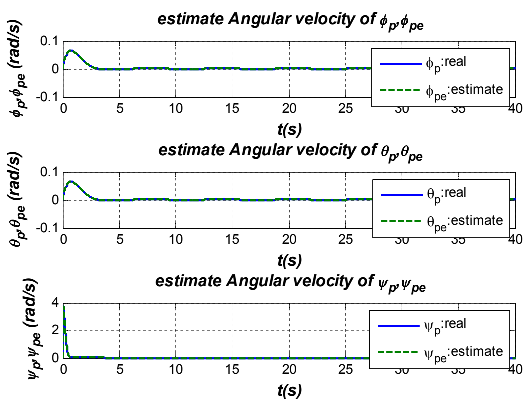

Fig. 8. Angular velocity results

Figure 6, represents the results of the real Euler's angles and their estimated, are tend to zero value after finished of the maneuverer.

Figs. 7 and 8, show the Linear velocity and the angular velocity results respectively, and notice the efficiency of the observer.

Fig. 9, illustrates the tracking errors with presence the wind disturbance, we notice that the tracking is effective, and the errors which vanish after a finite time with a perfect convergence. But the error along z direction is tend to constant value near to zero because of the presence of disturbance, this which confirm the result in Fig. 4. Fig. 10 and 11, represents the observer errors, of the position (x,y,z), and the linear velocity $(\dot{x}, \dot{y}, \dot{z})$ of the X4-flyer. We notice that the observation is good since the error tend to zero, This signifying the perfect convergence. Fig. 12, shows the perfect convergence and following of desired trajectory by the real one and the observer states the evolution of the Drone and its stabilization in 3D displacement.

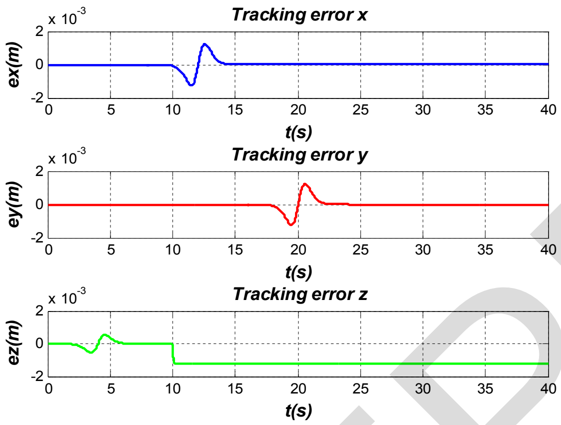

Fig. 9. Tracking errors

V.2. Sliding Mode Observer

It is concluded from the simulations, made without wind disturbance, that the sliding mode observer (SMO) gives satisfactory results.

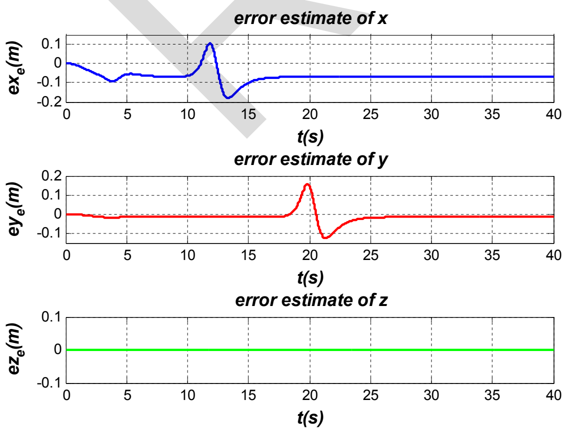

Fig. 10. Observation errors (x,y,z)

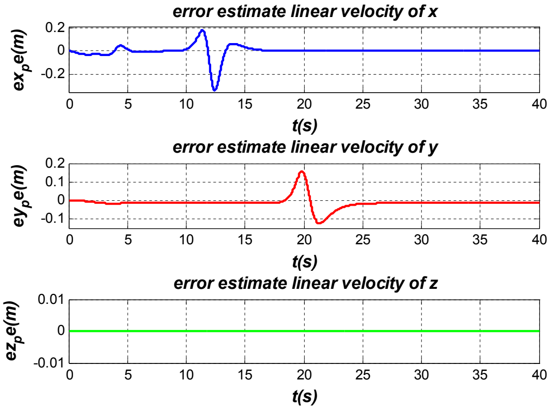

Fig. 11. Observer error for linear velocity

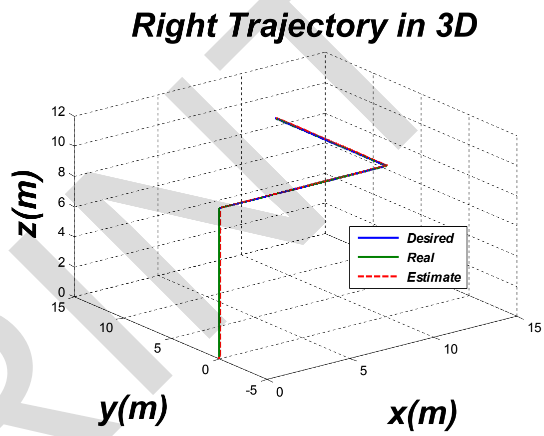

Fig. 12. Tracking trajectory in 3D

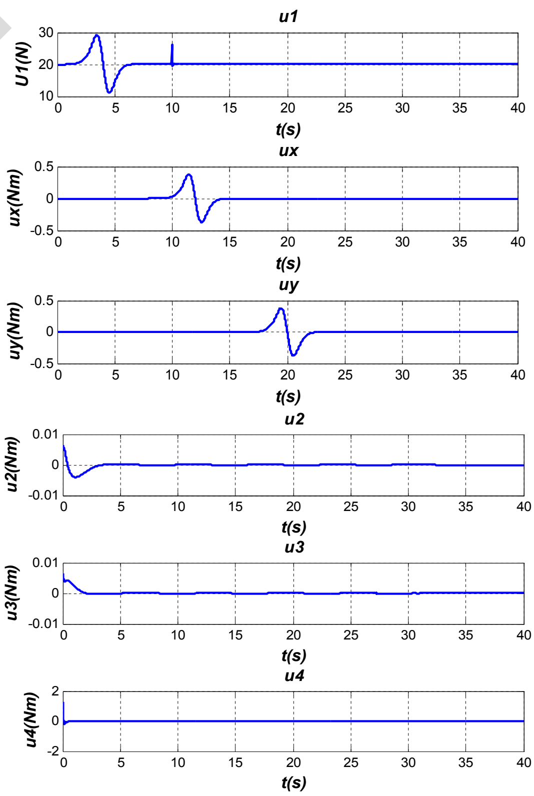

Fig. 13. Backstepping control signals with wind disturbance (6N)

The results of estimation errors given in Figs. 20 and 21 of position and linear velocity respectively show the efficiency of the observer, and notice the sliding mode observer give the beset result compared with a high gain observer (HGO). Fig. 19 see the tracking errors which vanish after a finite time with a perfect convergence.

When wind disturbances are introduced the Figs. 14, 15 and 19 reflect the robustness of the close loop observer–controller, and the perfect convergence of the real and estimate states given by (SMO) compared with (HGO). Figs. 16, 17 and 18, are present the real and estimate Euler's angles, Linear velocity and the angular velocity results respectively, we notice the beset convergence and the efficiency of the observer (SMO). Fig. 22, illustrate. The 3D displacement with straight connection, we notice the good convergence of the real and the observer states to desired trajectory.

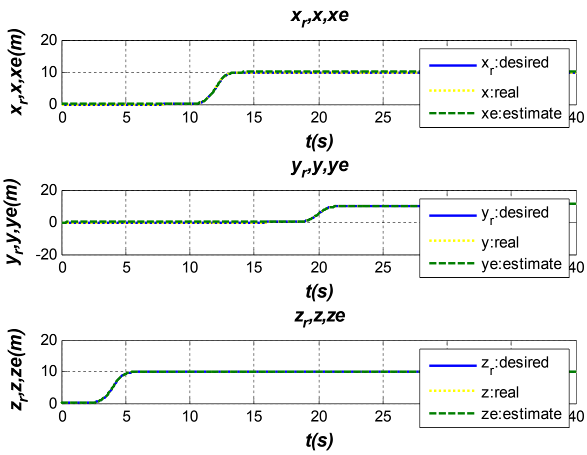

Fig. 14. Tracking simulation results with wind disturbance (6N)

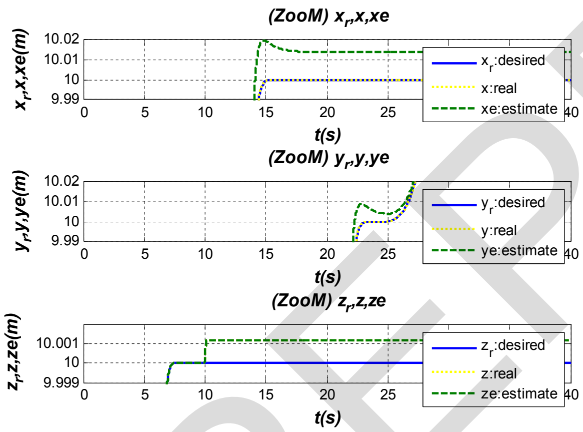

Fig. 15. Tracking simulation results Zoom with wind disturbance (6N)

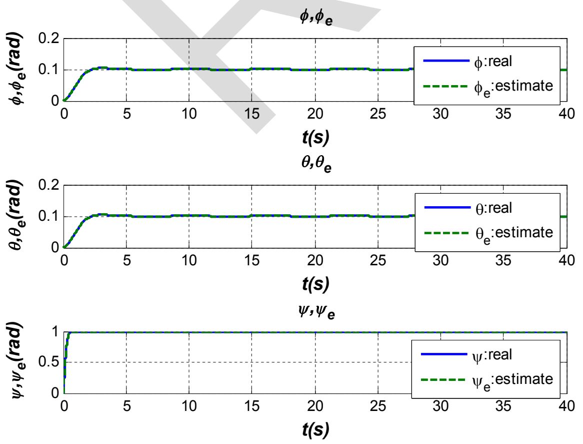

Fig. 16. The pitch (θ) roll (ϕ) and yaw (ψ) angles.

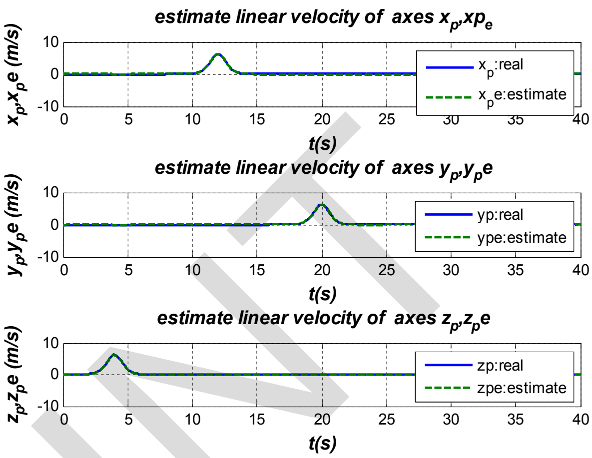

Fig. 17. Linear velocity results

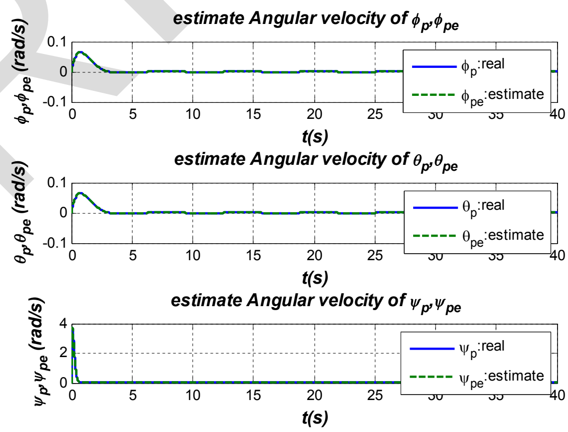

Fig. 18. Angular velocity results

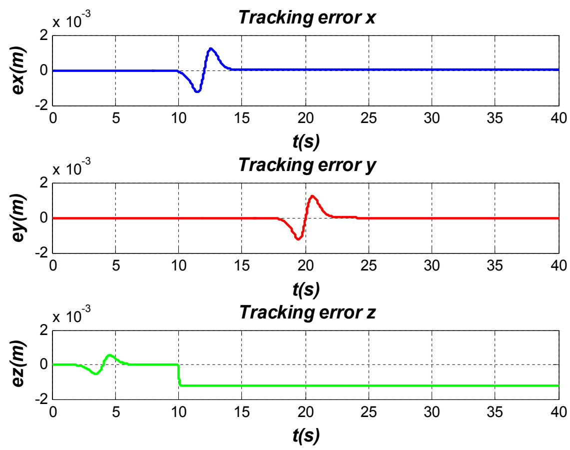

Fig. 19. Tracking errors

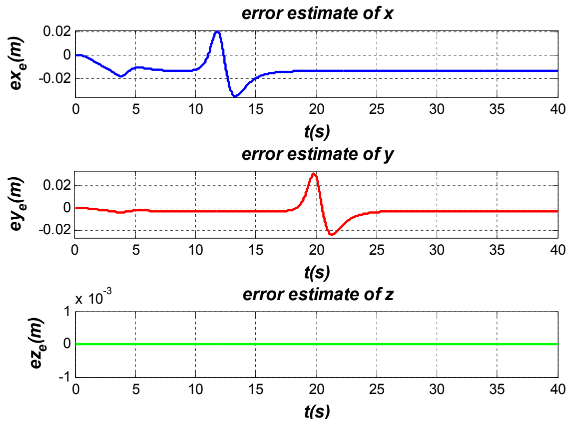

Fig. 20. Observation errors (x,y,z)

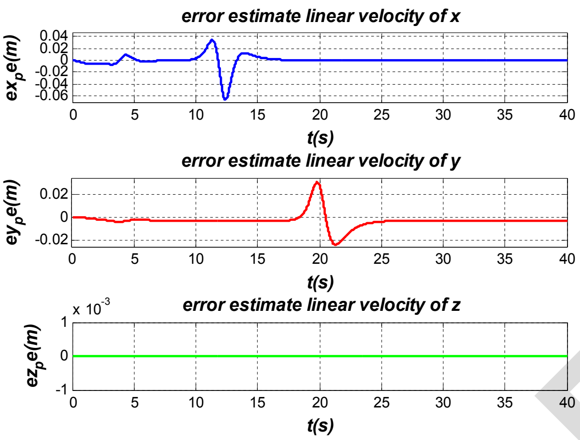

Fig. 21. Observer error for linear velocity

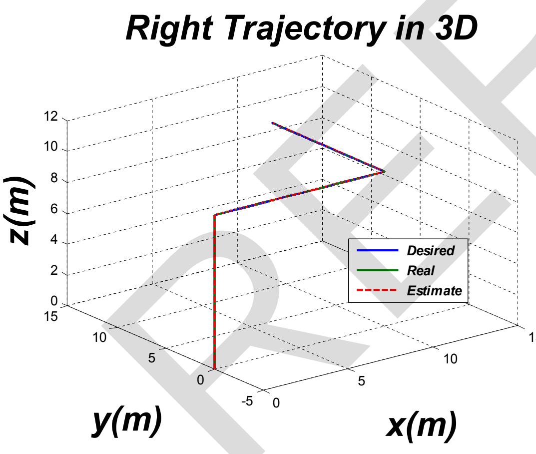

Fig. 22. Tracking trajectory in 3D

VI. Conclusion

In this paper, we studied a dynamic modeling of flyer engine called X4-flayer. We have considered in this work the stabilizing/tracking control and the observation problem for the three decoupled displacements of the engine. The objectives are to test the capability of the two observers called sliding mode and high gain observer. A comparison between SMO and HGO is given and discuss.

The use of an observer sliding mode in the Backstepping control of quadrotor seems simple and effective, with their robust appearance, the sliding mode bring to scheme of control the robustness via the observer through a rapid convergence and guarantee estimated states to real states, the command completes the objectives of stability and performance robustness with the estimated states.

Like prospects to be approached in the future, the consideration of the problem of asymmetry in the structure of the quadrotor which can considerably deteriorate the stability and the performances of any even robust order, of the moment which count does not hold any. The higher order sliding mode observer seems also very interesting to apply to a system such as the quadrotor.

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