

# International Review of Physics (IREPHY)

Contents:

- A Studying of Some Physical and Mechanical Properties  
for Hybrid Particular Composites Polyester** 1  
*by Kareem A. Jasim, R. S. Al-Khafaji*
- Numerical Tribute to Achievements of Euler** 5  
*by Carlos Figueroa, Lamberto Castro, Julio C. Campos*
- Binder Cumulant for Bond Diluted Ising Model** 15  
*by I. Zergoug, R. Bouamrane*
- Effect of Laser Pulse Energy and Vacuum Pressure  
on the Cu Laser Induced Plasma Plume Length** 19  
*by Kadhim A. Aadam, Ali A-K. Hussain, Mohammed R. Abdulameer*
- Dielectric and Microwave Properties of Natural Rubber Based Composites  
Containing Fullerene Black** 23  
*by Omar A. Al-Hartomy, Ahmed Al-Ghamdi, Nikolay Dishovsky, Desislava Slavcheva, Vladimir Iliev,  
Farid El-Tantawy*



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# Binder Cumulant for Bond Diluted Ising Model

I. Zergoug, R. Bouamrane

**Abstract** – The fourth order Binder cumulant  $U$  and the critical Binder cumulant  $U^*$  of bond-diluted Ising model on a square lattice are studied using Monte Carlo techniques. The values of the transition temperature  $T_c$  obtained from  $U^*$  for different probabilities of bond dilution  $p$  are discussed. When  $p$  is not too large ( $p < 0.35$ ),  $T_c$  decreases when  $p$  increases and  $U^*$  also decreases. For  $p > 0.35$ , things are not so clear and our results show that the determination of  $T_c$  from  $U$  is questionable. **Copyright © 2014 Praise Worthy Prize S.r.l. - All rights reserved.**

**Keywords:** Diluted Ising Model, Binder Cumulant, Monte Carlo Simulation

## Nomenclature

$U$	Binder cumulant
$U^*$	Critical Binder cumulant
$p$	Concentration of dilution
$T_c$	Critical temperature of transition
$L$	Lattice size
$H$	Hamiltonian
$J$	Exchange coupling constant
$S$	Spin value
$N$	Total number of spins
$k_B$	Boltzmann constante
$M$	Magnetization
$E$	Energy
$C_v$	Specific heat

## I. Introduction

The Binder cumulant  $U$ , the fourth order cumulant of the order parameter [1], plays an important role in the field of phase transition and critical phenomena. It is well known that the determination of the transition phase point is obtained by locating the intersection of the cumulant function of temperature at different sizes.

The cumulant may also be used to compute the critical exponent of the correlation length, and thence identify the universality class of the transition.

$U^*$ , the value of the Binder cumulant at the transition temperature  $T_c$ , has received a lot of interest. The numerical value of critical binder cumulant has been determined very accurately in the case of two dimensional Ising model with ferromagnetic nearest neighbour, using Monte Carlo techniques [2], [15] and transfer matrix methods augmented by finite size extrapolations to the thermodynamic limit [3], [16]. In the case of square lattice with periodic boundary conditions, the resulting value is  $U^* = 0.61069$  [3].

In fact, different values of  $U^*$  have been obtained when considering various boundary conditions, lattice shapes, and lattice structures (anisotropic interactions)

[1], [3], [4], [5], [9]-[14]. For instance, with free boundary conditions, the critical cumulant of nearest neighbour Ising model on square and triangular lattice are respectively  $U^* = 0.396 \pm 0.002$  and  $U^* = 0.379 \pm 0.001$  [5]. In the present paper, we report the value of the critical Binder cumulant in the case of bond diluted Ising model on a square lattice with free boundary conditions.

The paper is organized as follows: in the next section, the model and the Monte Carlo method are introduced.

Then, simulation results are presented and finally, results are shown and discussed.

## II. Model and Method

The Hamiltonian of the model considered here reads [6]:

$$H = \sum_{\langle i,j \rangle} J_{ij} S_i S_j \quad (1)$$

where  $S_i = \pm 1$  is the spin at the site  $i$ . The exchange coupling constant  $J_{ij}$  is 0 for non-nearest-neighbours and 1 or 0 with probabilities  $p$  or  $(1-p)$  respectively. Free boundary conditions are used, and the regular square lattice size is  $L = \sqrt{N}$  (where  $N$  is the total number of spins in the system).

We have calculated the Binder cumulant at the phase transition  $T_c$  for different probability of bond dilution  $p$  (concentration of dilution). The exact critical temperature  $T_c$  for the regular square lattice is given by [7]

$$k_B T_c / J = 2 / \ln(\sqrt{2} + 1) = 2.26918\dots$$

The binder cumulant, the fourth-order cumulant of the magnetization, as usually defined, is [1]

$$U(T, L) = 1 - \frac{\langle M^4 \rangle}{3 \langle M^2 \rangle^2} \quad (2)$$

where  $\langle M^2 \rangle$  and  $\langle M^4 \rangle$  denote thermal averages of the second and fourth moments of the magnetization. In the Ising model case, the cumulant  $U(T,L)$  approaches the value  $2/3$  at temperature  $T < T_c$  and the value  $0$  at  $T > T_c$ , reflecting the Gaussian form of the magnetization distribution [1]. At  $T_c$ , the Binder cumulant  $U^*$  acquires a non-trivial value, which will be estimated in this paper for different  $p$  dilution value. To achieve this, we performed Monte Carlo simulation [8] using standard single spin flip algorithms. We divided the equilibration process into "timesteps", a Monte Carlo step per spin.

The measurement of the thermodynamic quantities is carried out after a certain number of timesteps which we name a "timeslice". To estimate accurately the fluctuations of thermodynamic quantities, measurements are taken at each timeslice and averaged over a larger number of samples. When the difference of the averaged energy between two successively timeslices is lower than a certain given value, the simulation is stopped and various thermodynamic quantities are obtained. This process is repeated for several temperature values  $T$ .

As usual, the energy  $E$  and the specific heat  $C_v$  are obtained by:

$$E = -\frac{1}{N} \sum_{\langle i,j \rangle} J_{ij} S_i S_j \quad (3)$$

$$C_v = N\beta^2 \left( \langle E^2 \rangle - \langle E \rangle^2 \right) \quad (4)$$

where  $N$  is the total number of spins in the system,  $\beta=1/k_B T$  ( $T$  is the temperature and  $k_B$  is the Boltzmann constant taken to unity here).

### III. Results and Discussion

We consider nearest neighbour Ising model on the square lattice with free boundary conditions. The calculations were carried out for  $p = 0.0, 0.05, 0.10, 0.15$  and  $0.20$  using lattice sizes  $L = 256, 1024$  and  $4096$ . 10000 samples are used for averaging.

First of all, thermodynamic quantities are plotted in Fig. 1 and Fig. 2 for  $L = 4096$  and for various concentration of dilution.

A calculation is added in case of  $p = 0.5$  for lattice size  $L = 512, 1024$  and  $4096$  with 16384 averaging samples for the two first sizes, 590000 averaging samples for the last one. From Fig. 3, we can get the value of the critical temperature  $T_c$  for different concentration of dilution  $p$  from the intersection of the Binder cumulant for different lattice sizes. The critical temperature  $T_c$  are shown decrease linearly as the dilution  $p$  increases (see Fig. 4).

We focus now on the study of the critical binder cumulant  $U^*$ . In previous simulation [5], for isotropic nearest neighbour Ising model on square lattice and free boundary conditions the value of  $U^* = 0.396 \pm 0.002$  has been found.

The author claimed that the critical cumulant depends only on the shape not on the lattice structure.

In our case, we study the critical binder cumulant  $U^*$  dependence as a function of the dilution  $p$  (see Fig. 5).

For  $p = 0$  we get  $U^* = 0.4022 \pm 0.0007$ . The value of  $U^*$  decreases linearly the increase of the dilution  $p$ . An interesting results is shown for  $p = 0.5$ . On Fig. 6 and Fig. 7 is shown the variation of the Binder cumulant  $U$  and the specific heat  $C_v$  against the temperature respectively.

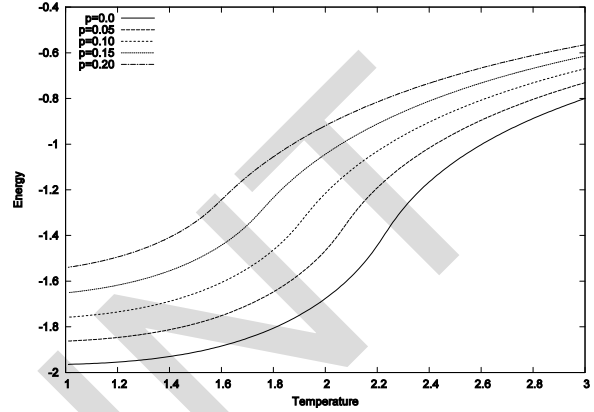


Fig. 1. The energy  $E$  as a function of the temperature for different concentration of dilution  $p$

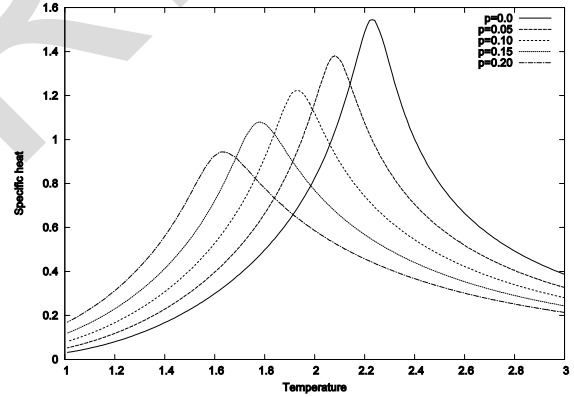


Fig. 2. The specific heat  $C_v$  as a function of the temperature for different dilution  $p$

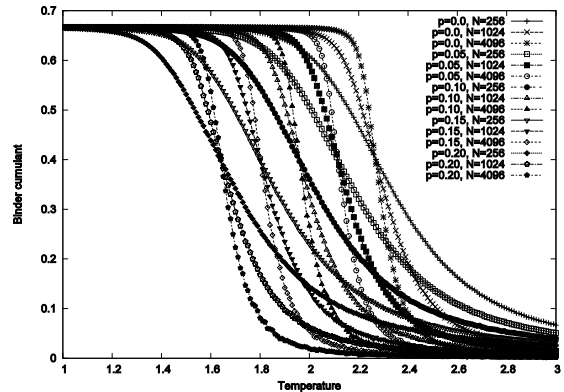


Fig. 3. The Binder cumulant  $U$  versus the temperature for different dilution  $p$

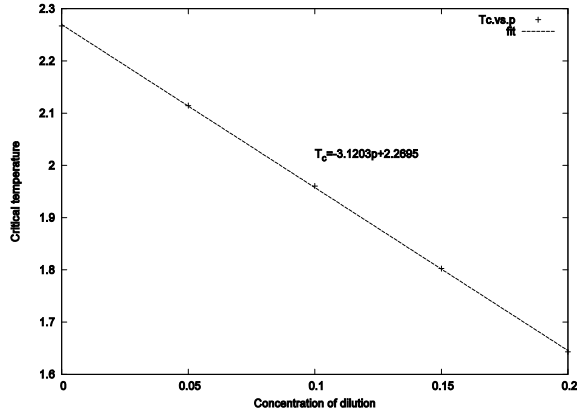


Fig. 4. Variation of the critical temperature  $T_c$  versus dilution  $p$

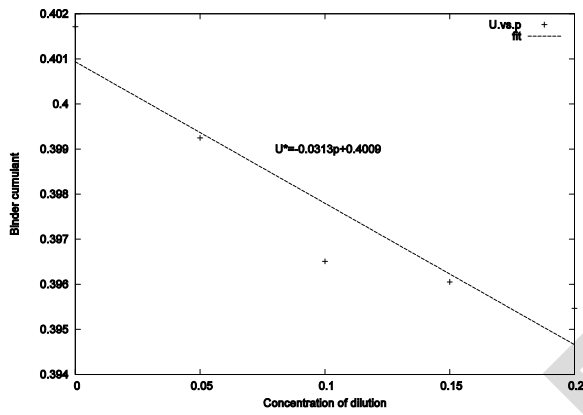


Fig. 5. Variation of the critical Binder cumulant  $U^*$  versus dilution  $p$

It is comment practice that the value of the critical temperature  $T_c$  is obtained from intersecting critical Binder cumulant ( $U^*$ ) curves for various system sizes. Also,  $T_c$  normally corresponds to the maximum value of the specific heat  $C_v(\max)$ . Using the first method, 590000 averaging samples are used for  $p=0.5$  and the intersection of  $U(T)$  curves for different lattice sizes yields the value  $T_c = 0.39$  (see Fig. 6). Moreover, if we seek  $T_c$  from locating the maximum of  $C_v(T)$ , we get  $T_c = 1.03$  (see Fig. 7).

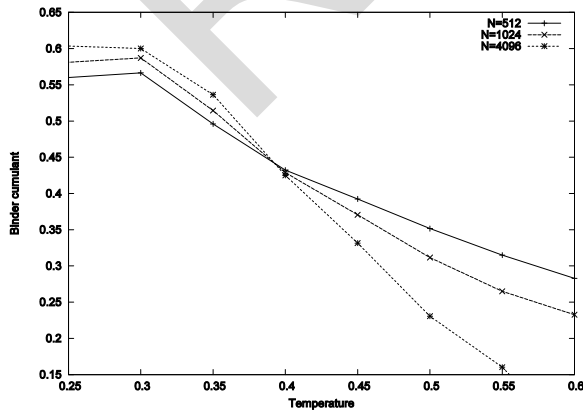


Fig. 6. Variation of the Binder cumulant  $U$  versus the temperature for  $p = 0.5$

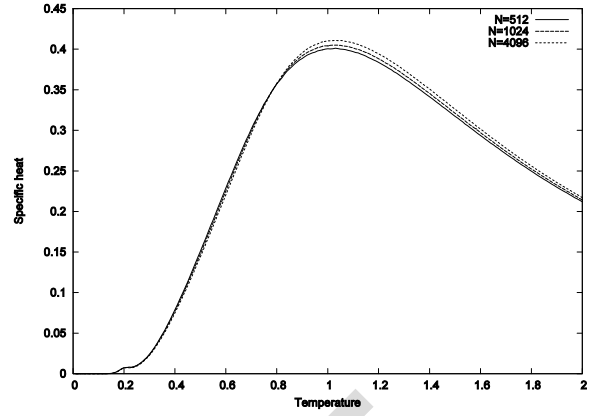


Fig. 7. Variation of the specific heat  $C_v$  versus the temperature for  $p = 0.5$  for different lattice sizes

It appears that the Binder cumulant method of determining critical temperatures breaks down as the bond concentration  $p$  approaches the bond percolation threshold ( $p = 0.5$ ).

The difference between the two methods in the determination of the value of  $T_c$  exceeds 1% for  $p > 0.35$ . Table I shows values of  $T_c$  obtained by the two methods for various  $p$ .

For  $p \leq 0.30$ ,  $T_c$  (from  $U$ ) is slightly larger than the  $T_c$  (from  $C_v$ ) which can be explained from finite scaling law. But for  $p \geq 0.35$ ,  $T_c$  (from  $U$ ) is quite smaller than  $T_c$  (from  $C_v$ ) which cannot be resolved by scaling laws.

TABLE I  
THE VALUES OF  $T_c$  DETERMINED FROM THE TWO METHODS FOR DIFFERENT BOND CONCENTRATION  $P$

$p$	$T_c$ (From $U$ )	$T_c$ (From $C_v$ max)
0.0	2.2670	2.2306
0.05	2.1145	2.0813
0.10	1.9605	1.9325
0.15	1.8024	1.7804
0.20	1.6430	1.6360
0.25	1.4817	1.4914
0.30	1.3163	1.2930
0.35	1.1342	1.2515
0.40	0.9252	1.1438
0.45	0.6733	1.1029
0.50	0.3924	1.0365

## IV. Conclusion

The critical Binder cumulant  $U^*$  for two dimensional bond diluted Ising model on square lattice with free boundary conditions has been studied. The simulation findings show that the critical binder cumulant depends on the concentration of dilution  $p$ . The value of  $U^*$  for  $p = 0$  was equal to  $0.4022 \pm 0.0007$  with respect to [5] that was  $0.396 \pm 0.002$ .

The determination of the critical temperature  $T_c$  using the Binder cumulant must be use carefully for disordered systems.  $T_c$  obtained from  $C_v(\max)$  is more likely reliable for strongly disordered systems. Further investigations are being done with larger system sizes and larger samples to get more precisely in order to settle this issue.

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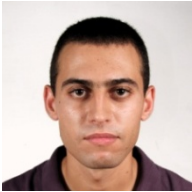
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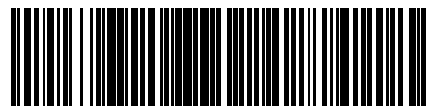
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