People's Democratioc Republic of Algeria
Ministry of Higher Education and Scientific Research
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Faculty of Physics
Department of Materials Technology

COURSE HANDOUT

REGULATION AND CONTROL SYSTEM

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Preface

This document is intended for master's students in Physics, specializing in Energy Physics and Renewable Energy, as part of the official curriculum. The primary objective of this module is to master the fundamentals of studying control and regulation systems, which are technical disciplines aimed at analyzing and designing practical control systems and other technological devices. The objectives of this handout are to help students become familiar with control components, the principles of control and regulation, as well as the analysis and synthesis of controlled systems.

The content of this handout is structured into seven chapters:

- Chapter I: Basic notions of regulation and control systems
- Chapter II: Reminders about the Laplace transform
- Chapter III: Modeling of linear systems
- Chapter IV: Temporal response of linear systems
- Chapter V: Stability of servo systems
- Chapter VI: Frequency analysis of linear systems
- Chapter VII: Accuracy of servo systems

Each chapter has been reinforced by a series of exercises with their answers, to deepen the understanding of the lesson.

This document was produced with the aim of ensuring relatively uniform and coherent teaching and for didactic purposes, in order to try to identify the essential, simplified ideas, as well as the main bases assigned to this module.

We hope that this work will be profitable and will serve as a reference for anyone interested in the study of regulation and controlled systems.

Contents

Chapter I:

Basic notions of regulation and control systems

I. Basic notions of regulation and control systems

In the following section of the chapter, we will recall the basic notions of regulation and control.

I.1 Definition of regulation

-Regulation makes it possible to maintain a physical quantity at a constant value regardless of external disturbances.

The overall objective of regulation can be summarized by these three key words: Measure, Compare and Correct.

-Regulation consists of automatically maintaining a physical quantity at the desired value regardless of external disturbances.

-The regulation of a quantity is commonly called: Process regulation.

Examples:

- Regulation of the level of a tank.
- Regulation of the temperature of an oven.
- Motor speed regulation.
- Regulation of flow in a pipe.
- Golf cart speed regulation.

I.1.1 Physical quantities: Most industrial processes require controlling a certain number of parameters (physical quantities).

I.1.1.1. Type of physical quantities: Physical quantities can be classified into several families, primarily based on their nature and how they are defined:

- **Thermal:** (temperature, thermal sensor, thermal flow).
- **Electrical:** (current, voltage, charge, impedance, dielectric).
- **Mechanics:** (displacement, force, mass, flow, etc.).
- **Magnetic:** (magnetic field, permeability).
- **Radiative:** (visible light, X-ray, microwave).

- **Biochemical:** (humidity, gas, hormone, molecule).

I.1.1.2. Examples:

Level regulation: Liquid level in a tank.

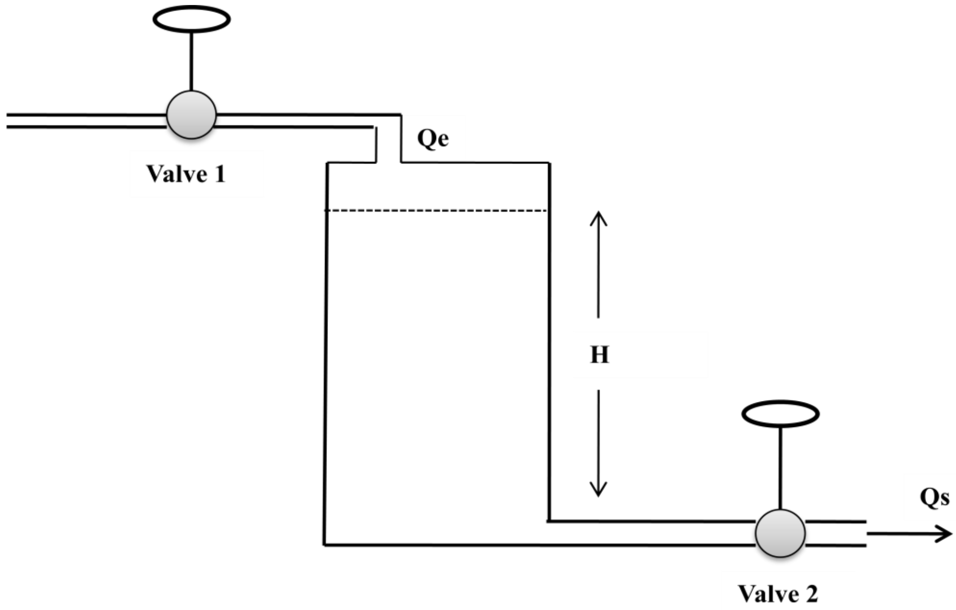


Figure I.1: Level regulation

Our objective is to maintain a constant H level: level regulation.

-To keep the level constant, the liquid entering through valve 1 must at all times be equal to that exiting through valve 2; valve 1= valve 2.

Input quantities: the quantities which modify the state of the system

• Qe: inlet (feed) flow

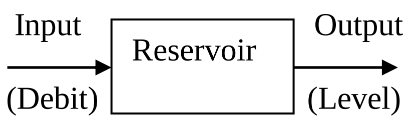
• Qs: outlet flow (withdrawal flow)

Output quantities: The quantities which characterize the state of the system

• H**:** The level.

-In automatic vocabulary, the liquid level is called the quantity to be controlled or Output.

-The opening of valve 1 or the flow rate in 1 is called the control quantity or Inlet.



I.1.2. Terminology and concepts

1.1.2.1. Manual regulation (manual control)

The modification to the controlled variable can be carried out by an operator continuously observing the controlled variable by modifying the controlling variable accordingly. In this case, we are in the presence of a manual control.

Taking an example of level regulation in a tank supplied by a random flow.

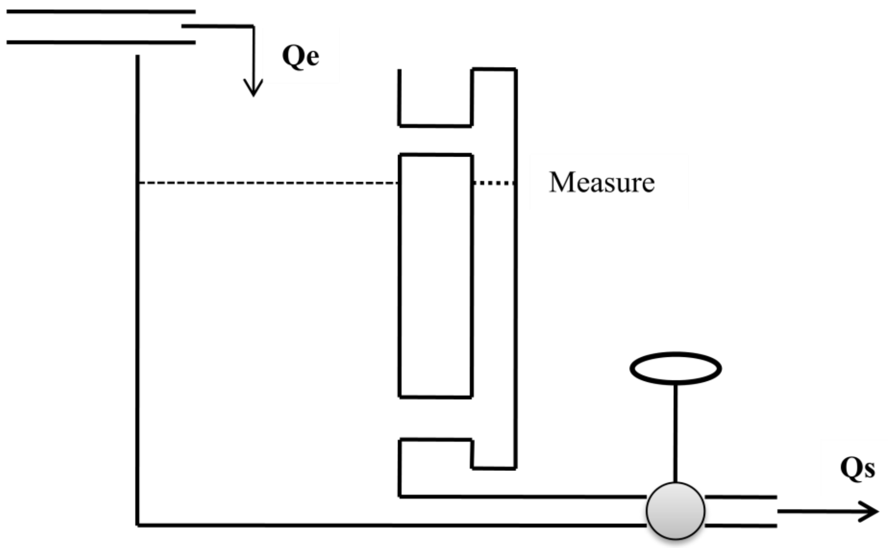


Figure I.2: Regulation with manual valve

1.1.2.2 Automatic regulation (automatic control)

In this case, the measurement of the regulated quantity and the modification of the regulating quantity are carried out automatically by means of devices called regulators in which a control law (algorithm) is implemented. In automatic regulation there is therefore no intervention by a human operator.

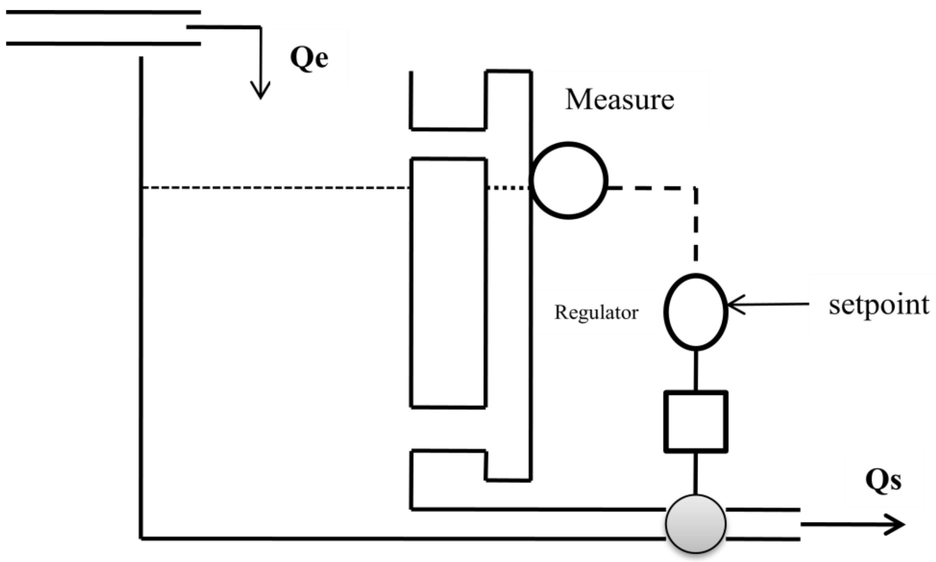


Figure I.3: Regulation with automatic valve

The operator (the regulator) constantly compares the measurement M and the setpoint (reference) C and acts on the valve in such a way that the difference ($M - C$) cancels out.

- If the measurement value M is greater than the setpoint C, it means that (M - C) is positive, so the valve should be opened.
- If the measurement value M is less than the setpoint C, this means that (M - C) is negative, and therefore the valve must be closed.
- If the measured value M is equal to the setpoint C, then (M - C) is zero. This therefore no longer affects the valve.

There must then be a disturbance in the system or a variation in the setpoint C so that (M - C) is no longer zero.

I.1.3 Objectives of regulation

To regulate a quantity is to obtain from it a given behavior, in an environment likely to present variations.

Automatic systems actually provide two types of functions:

- Maintain the ordered quantity, or regulated quantity, at a reference value despite variations in external conditions; we talk about regulation

- Responding to changes in objective, or to a variable objective such as the pursuit of target, we are talking about a servo function.

I.2 Enslavement

I.2.1 Definition of a system:

A system is a set of material elements acting and reacting on each other, and it is characterized by the relationship that exists between one or more input quantities and one or more output quantities.

The simplest representation of a system is as follows:

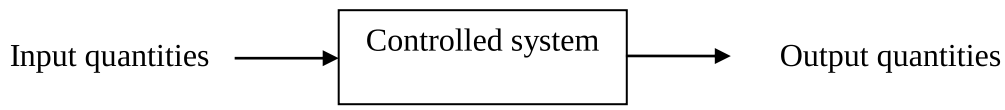
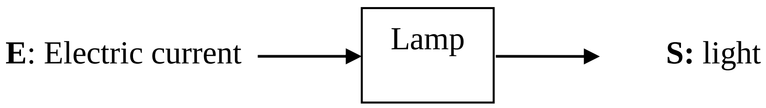


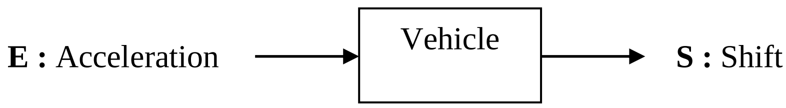
Figure I.4: Open loop system

Examples:

1- **A lamp:**



2- **Vehicle:**



3- **Television:**

1.2.2 Input and output quantities:

1.2.2.1. Input quantities: They are independent of the system and influence its state.

1.2.2.2. Output quantities: They are produced by the system, and they depend on the latter, evolving under the action of the input quantity.

1.2.2.3 Secondary inputs (disturbances): In reality, any system can be affected by disturbing quantities that hinder the proper functioning of the system.

Example: heating a room: opening then closing the door or a window.

I.2.3 Slave systems

- A servo system is a so-called follower system; it is the setpoint that varies. Example: a machine tool which must machine a part according to a given profile, a missile which pursues a target.

- A servo-controlled system is a looped system in which the return quantity is compared to the input quantity by producing a signal, called deviation. This deviation signal is adapted and amplified in order to control the operational part.

Among the controlled systems, we distinguish between regulator systems and tracking systems

- We speak of a regulator system when we want the output to take a precise value equal to a fixed input setpoint.
- We speak of a tracking system when we want the output to follow an input instruction that varies over time and whose evolution is not always known in advance.

I.2.4 Nature of input and output signals:

The input and output signals of a system are functions of time: if at each moment their amplitudes are perfectly known, the signal is said to be deterministic (example: unit step, sinusoid, etc.).

If, on the other hand, at each moment, we only know the probability of the signal having this or that amplitude, we say that it is random (e.g., noise).

I.2.4.1 Signal: Physical quantity generated by a device or translated by a sensor (flow temperature, etc.)

I.2.4.2 Input signal: independent of the system, it is broken down into controllable and non-controllable (disturbances)

I.2.4.3 Output signal: system-dependent and input signal. We distinguish between observable and non-observable output.

I.3 Concept of system, Open Loop (BO), Closed Loop (BF):

I.3.1 Open loop regulation

An open-loop system is a system that does not have a feedback chain between the output and the input. The control of this type of regulation is developed without knowing the output quantities. The open-loop system block diagram is given in the figure below.

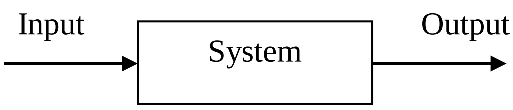


Figure I.5: Open loop system

Open-loop regulation has the advantages of anticipating phenomena and obtaining very short response times, eliminating the risks of pumping, but it is also the only possible solution when there is no final control possible.

Among its disadvantages is the need to know the law governing how the process works. Also, the absence of means to control and compensate for possible errors, drifts, and accidents that may occur inside the loop.

I.3.2 Closed loop regulation

The closed loop is a system that includes a feedback chain between the output and the input. It is capable of stabilizing an unstable system in open loop. The block diagram of a closed-loop system is given in the figure below.

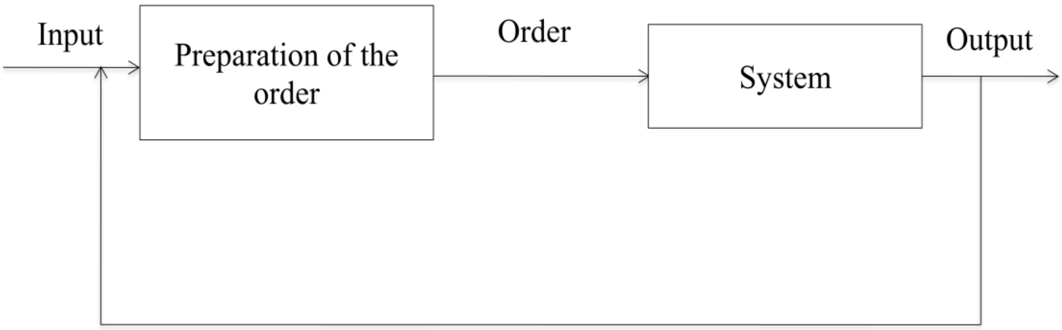


Figure I. 5: Closed loop system

The closed loop allows the compensation of a large part of the disturbing factors by counter-reaction (feedback). Another advantage of the closed loop is that precise knowledge of the laws and behavior of the different components of the loop is not necessary.

On the other hand, this type of regulation requires the precision of the measured values and the setpoint. More importantly, the wrong choice of certain components can generate the pumping phenomenon. Added to this, the failure to anticipate closed-loop regulation, which can sometimes be annoying.

I.4 Structure of a slave system

I.4.1 Action chain and return chain

A slave system is a looped system whose general structure is given as follows:

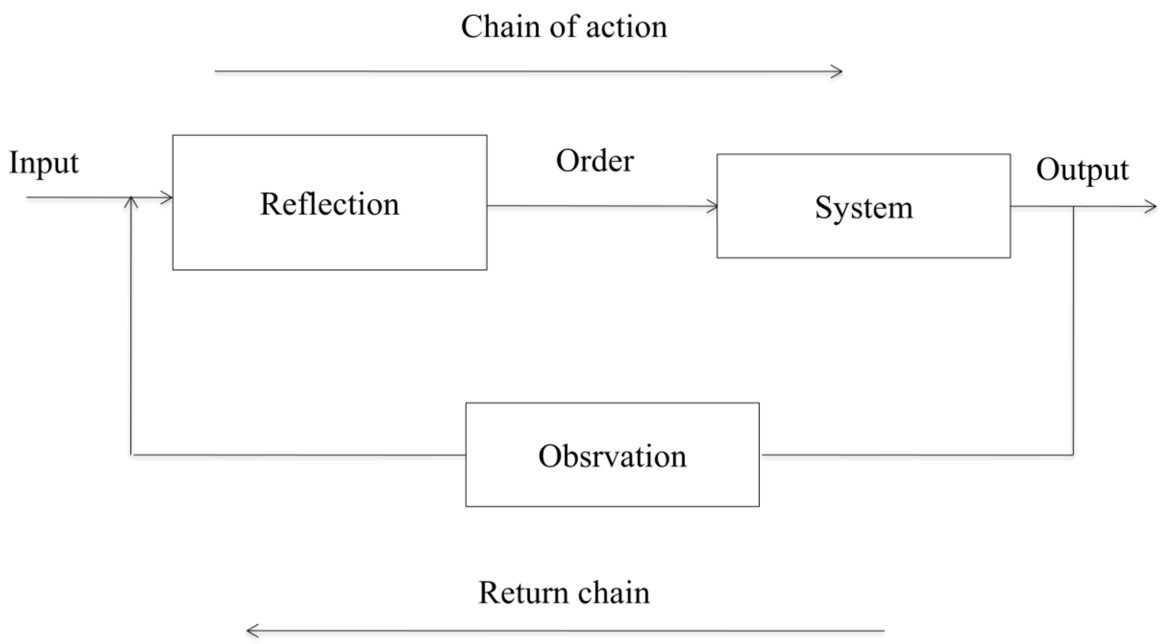


Figure I.6: Structure of a servo system

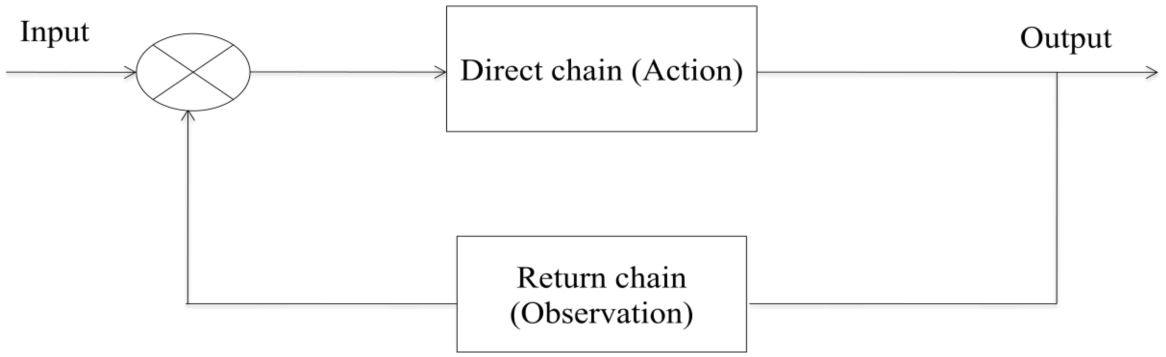


Figure I.7: General diagram of the control

I.4.2 Functional diagram of a control (block diagram)

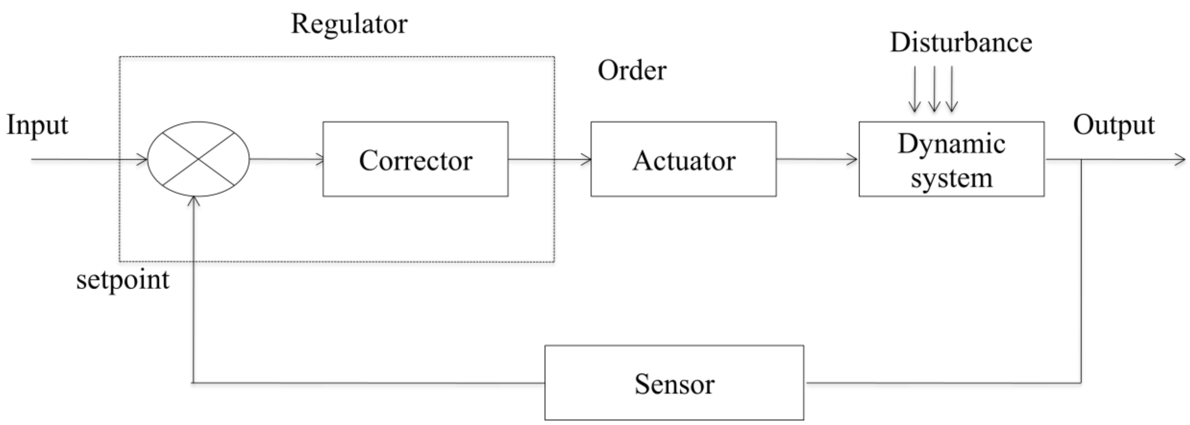


Figure I.8: Functional diagram of a control

- The regulator (comparator + corrector) develops the control order from the error signal ε: it is the intelligent organ.

- The actuator, or the action member, generally provides the power necessary for the realization of the task is the muscular organ.

- The dynamic system evolves according to the action following physical laws that are specific to its own; the output is generally a physical quantity that characterizes the task at hand realized; in addition, this output can fluctuate depending on unpredictable disturbances.

- The sensor delivers from the output a quantity characterizing the observation (sensor accuracy implies system accuracy).

I.5 Performance of a slave system

A good servo system must have three essential characteristics:

I.5.1 Precision

This is the ability of the system to get as close as possible to the setpoint value.

In other words, the exit must follow the entry in all circumstances.

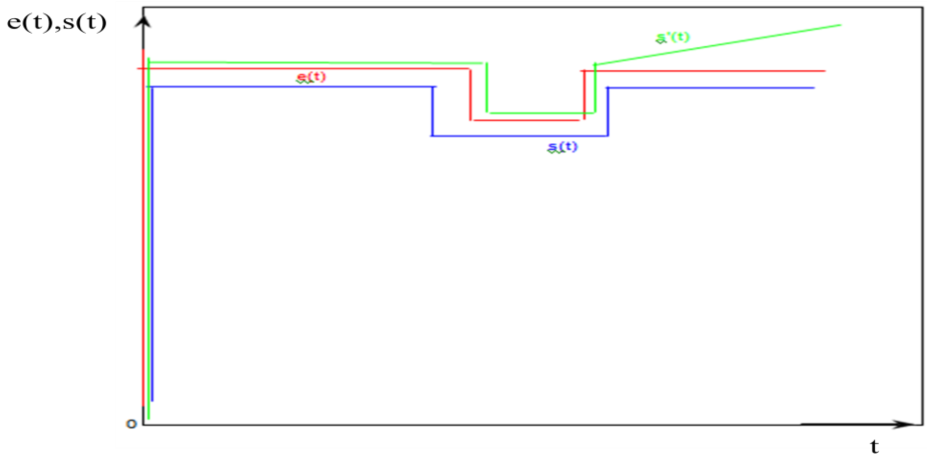


Figure I.9: Representation of a precise signal and a non-precise signal

Figure I. 6 represents two signals such that the signal s(t) is precise and the signal s’(t) is not precise.

I.5.2 Speed

Speed reflects the duration of the transient regime or the duration to reach the stable regime.

We call response time the time taken by the output (y(t) in figure (I. 10)) of the system to reach 95% of the final value V_f .

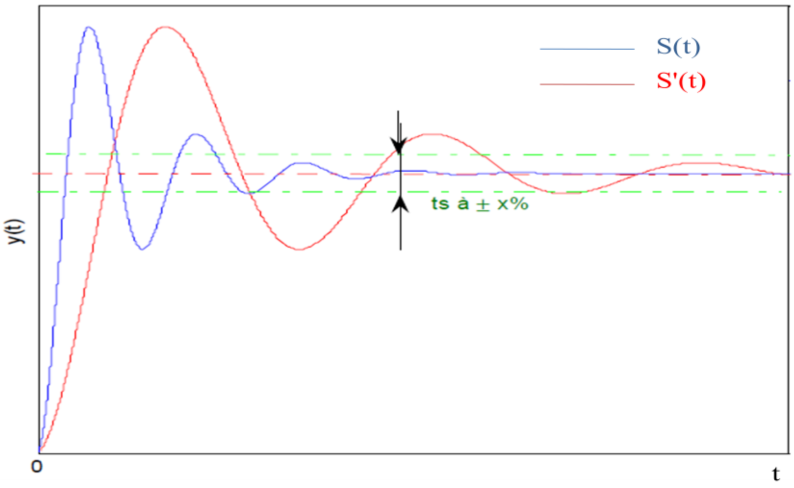


Figure I. 10: Representation of a fast signal and a slow signal

The signal s(t) is faster than the signal s'(t).

I.5.3 Stability

For a constant setpoint, the output must tend towards a constant output.

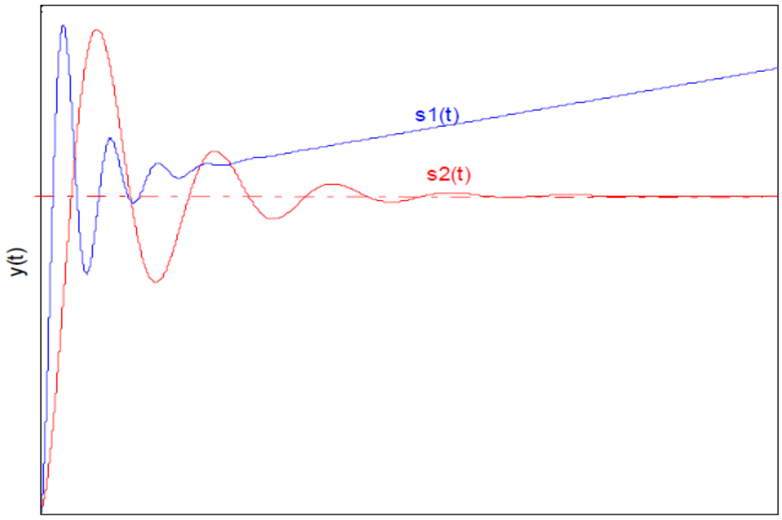


Figure I. 11: Representation of a stable signal and an unstable signal

The figure above represents two signals, of which the signal $S_2(t)$ is stable and the signal $S_1(t)$ is unstable.

Noticed:

A control system can perform two distinct functions:

Servo control: that is to say, the continuation by the output of a variable setpoint over time.

Regulation: that is to say, the compensation for the effect of variable disturbances on the output (the setpoint remaining fixed).

I.6 Correctors

The purpose of the correctors is to improve the performance of the slave system. We generally place this correction block in the direct chain, just at the output of the comparator. A corrector is a calculation algorithm which delivers a control signal based on the difference between the setpoint and the measurement.

The PID corrector acts in three ways:

- Proportional Action: the error is multiplied by a gain G
- Integral Action: the error is integrated and divided by a gain Ti

- Derived Action: the error is derived and multiplied by a gain Td

I.6.1 Proportional action (P)

The role of proportional action is to:

- Reduce response time
- Allows you to correct the effects of a disturbance
- Decrease the static error, but it does not cancel the error.
- Too much gain can destabilize the system.

I.6.2 Derivative action (D)

The role of the derived action is to:

- It allows you to “boost” the direct chain, therefore improving speed.
- But it does not cancel the static error.

I.6.3 Integral action (I)

The role of integral action is

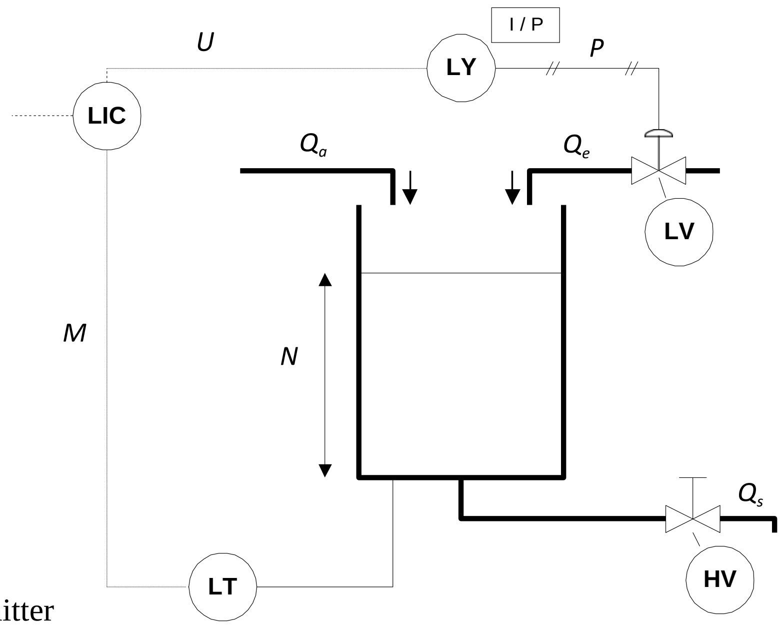
- Fixes the effects of a disturbance
- Cancels static error
- She's not very fast.

I.7 Solved exercises

Exercise 1

Automatic level regulation:

Level regulation of a tank of a syrup dilution installation: the aim is to maintain the level N constant in the tank to ensure a constant output flow Q_s regulated by the opening of a manual valve, the flow rate of recycled product Q_a not being constant.



LT: level transmitter

LIC: level indicator regulator

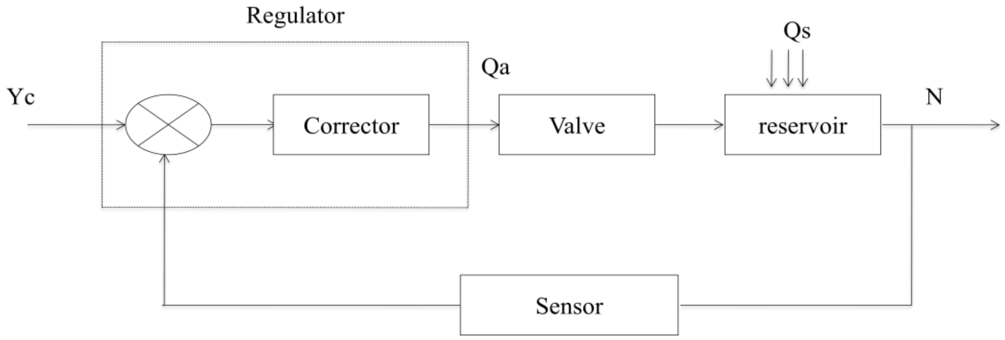
LY: current/voltage converter

LV: level regulating valve

HV: vanne manuelle

1- Establish the functional diagram of the level regulation loop.

Solution :

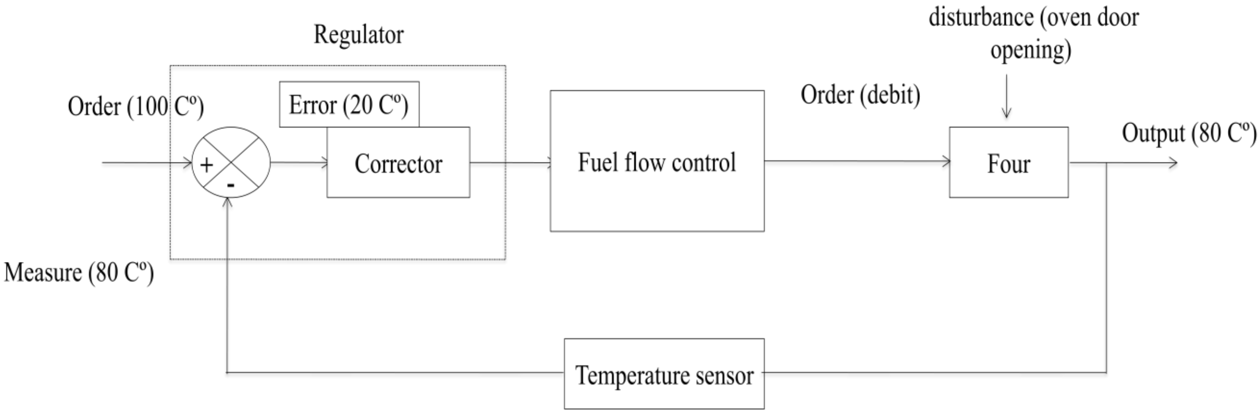


Exercise 2

Let's take the example of adjusting the temperature of an oven by acting on the flow of fuel, ensuring heat production. We will provide additional information to the regulator. This is to tell him the temperature of the oven. The controller compares the desired temperature (setpoint) with the actual temperature (measurement) to evaluate the deviation (error) and adjust accordingly (command).

Establish a Functional diagram of the temperature control loop.

Solution :



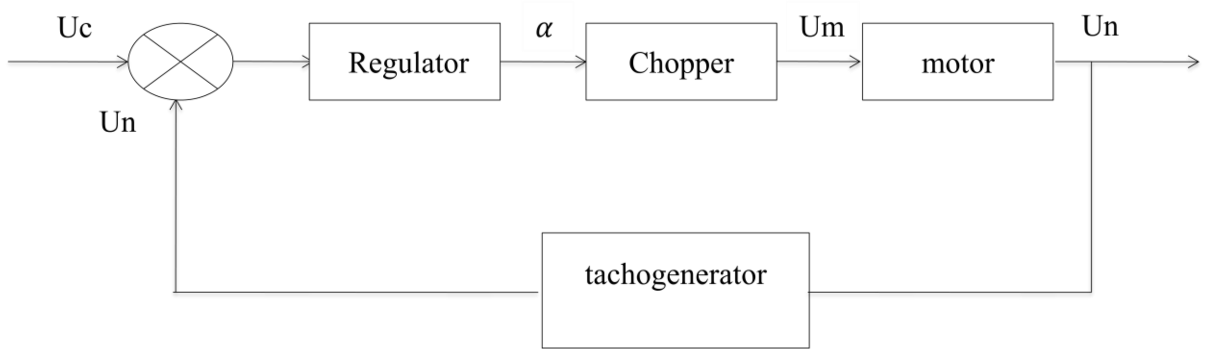
Exercise 3

A tachometric dynamo delivers a voltage Un, proportional to the speed of the engine. The comparator then makes the difference between the measurement voltage Un and the set voltage Uc. This difference is called deviation (ε) and then controls the direct chain. If the motor load decreases, the measured speed increases, the difference between the setpoint and the measurement decreases, the deviation decreases, the voltage at the motor terminals decreases, and the speed decreases. If the motor load increases, the measured speed decreases, the difference between the setpoint and the measurement increases, the deviation increases, the voltage across the motor terminals increases, and the speed increases.

- Create a functional diagram of this system.

Solution:

The functional diagram of this system is:



I.8 Additional exercises

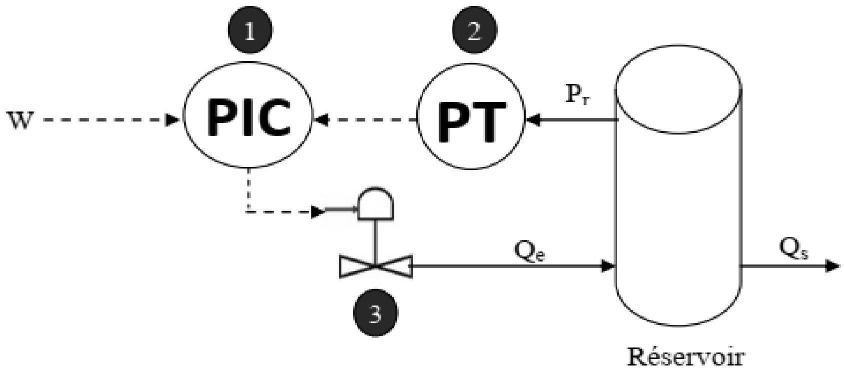
Exercise 1

We consider the heating system of an individual house consisting of an oil burner, a network of pipes and radiators, and a thermostat containing the measurement and control element**.**

1- Draw a block diagram for this control system.
2- Identify the main elements of the control loop.
3- Is the control problem of the servo or regulation type?

Exercise 2

We consider the following diagram representing pressure regulation in a tank:

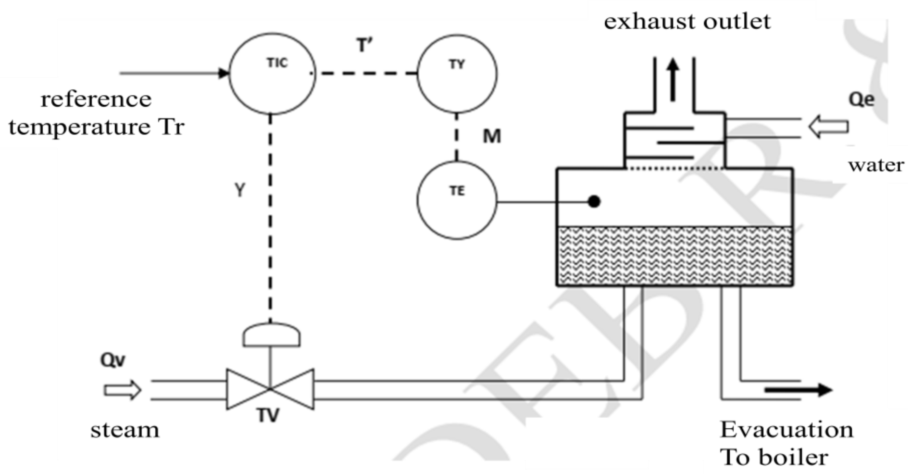


1- Define elements 1, 2, and 3.
2- Draw the regulation block diagram.

Exercise 3

Temperature regulation of a thermal degasser

A degasser is used to reduce the concentration of oxygen (O_2) and carbon dioxide (CO_2) in the water. It is used for the treatment of make-up water from industrial boilers. The reduction in oxygen levels and carbon dioxide reduces the risk of corrosion. Elimination is done by creating an atmosphere deprived of these gases on the intimate surface of the runoff water. In addition, the property of gases to be less soluble as the pressure is low and the temperature is high is used. To do this, the water contained in the degasser is maintained at a slight pressure (0.3 to 0.7 bar) and at the corresponding evaporation temperature (107 to 115 °C). It is thus in slight boiling (vaporization). The mixture of steam and gas released by the feed water is evacuated to the atmosphere through a vent (exhaust outlet) as it forms. The following figure represents the diagram for regulating the temperature set by acting on the steam flow.



TE: Probe for temperature measurement.

TY: Module for converting a voltage signal into a current signal.

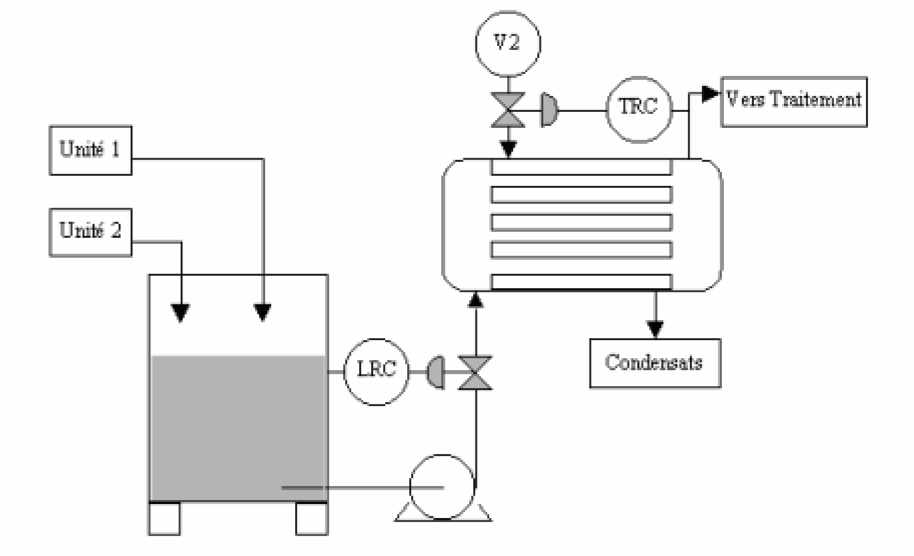
TIC: Industrial regulator composed of a comparator and a corrector.

TV: Flow adjustment valve equipped with a positioned

1- Find the functional diagram of this regulation loop.

Exercise 4

Or the following process:



The process fluid is the operating residue of units 1 and 2. The nominal flow rate is 600 L/h for unit 1 and 800 L/h for unit 2, with maximums of 1200 L/h for unit 1 and 1500 L/h for unit 2. The process fluid arrives in a storage tank 5.5 meters high; the nominal level of this tank is 4.5 meters. As the tank is located in a hall, the nominal temperature of the liquid stored is 25°C. The process fluid is taken up by a centrifugal pump and sent to a heat exchanger. The process fluid passes through the exchanger in the tubes, while the thermal fluid, saturated water vapor at 3 bar, condenses in the shell of the heat exchanger. The process fluid is thus heated to a nominal value of 80°C. It remains in a liquid state.

It should not be sprayed. Its boiling temperature is 140°C; it is potentially explosive. The nominal flow rate of water vapor required to heat the nominal flow rate of process fluid to a temperature of 80°C is 800 kg/h, a maximum of 2100 kg/h. The process fluid is then sent to a treatment unit.

1- For each of the 2 regulations, specify which the regulated, regulating, and disturbing quantities are. Give the setpoint value of each of them.

2- Choice of sensors: the LT is a passive sensor, with a scale range of 0 to 6.5 meters, 4-20 mA signal, equipped with intrinsic safety; the TT sensor is an active sensor, with a scale range of 10 to 160 °C, 4-20 mA signal, equipped with ADF security.

- Explain why these sensors are suitable.

Chapter II:

Reminders about the Laplace transform

II. Reminders on the Laplace transform

II. 1 Introduction

The Laplace transformation is an integral operation that allows you to transform a function of a real variable into a function of a complex variable. By this transformation, a linear differential equation can be represented by an algebraic equation. It also makes it possible to represent particular functions (Heaviside distribution, Dirac distribution, etc.) in a very elegant way. It is these possibilities that make the Laplace transformation interesting and popular with engineers. This transformation gave rise to the technique of operational calculation, or symbolic calculation, which facilitates the resolution of linear differential equations that will represent the systems that we are going to study.

II. 2 Definition

Consider a function F(t) such that F(t)=0 for t<0. We define its Laplace transform (TL) F(p) by:

$$F(p) = TL[f(t)] = \int_0^{+\infty} f(t).e^{-pt}.dt$$

We will admit that there exists a Laplace transform for all the functions that we will encounter. We will denote by lowercase letters the original functions (functions of time) and by uppercase letters the images (the functions of the variable p). In practice, the Laplace transforms will not be calculated, but we will use the transform table.

II. 3 Fundamental properties of the Laplace transformation

The following properties allow you to easily calculate (without using the definition of the Laplace transform) the Laplace transforms of certain functions.

II.3.1 Linearity

$$TL[a.f(t) + b.g(t)] = a.F(p) + b.G(p)$$

II.3.2 Derivation

$$TL\left[\frac{df(t)}{dt}\right] = p.F(p) - \lim_{t\to 0^+} f(t)$$

which is generalized:

$$TL\left[\frac{d^2\mathrm{f}}{dt^2}\right] = p^2.F(p) - P.\lim_{t\to 0^+} f(t) - \lim_{t\to 0^+} \frac{df(t)}{dt}$$

Often, f(t) and the successive derivatives of f(t) are zero at the initial time.

II.3.3 Integration

$$TL\left[\int_0^t f(\tau)d\tau\right] = \frac{F(p)}{p}$$

II.3.4 Delay

$$TL[f(t-\tau)] = e^{-\tau.p}.F(P)$$

II.3.5 Initial value theorem

$$\lim_{t\to 0^+} f(t) = \lim_{t\to +\infty} p.F(p)$$

II.3.6 Final value theorem

$$\lim_{t\to +\infty} f(t) = \lim_{t\to 0^+} p.F(p)$$

II.3.7 Translation of the Laplace variable

$$F(a+b) = TL[\,e^{-at}.f(t)]$$

II.3.8 Inverse Laplace transforms:

Just as a function of time can have a Laplace transform, it is possible from a function F(P) to find its original, in other words, the inverse Laplace transform:

$$f(t) = \int_{-\infty}^{+\infty} F(P)e^{\cdot pt}dt$$

II.4. Laplace transforms of some usual signals

II.4.1 Unit level:

The unit level (figure II.1) is the function u(t) such that:

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

We then have $u(p) = TL\big(u(t)\big) = \frac{1}{p}$

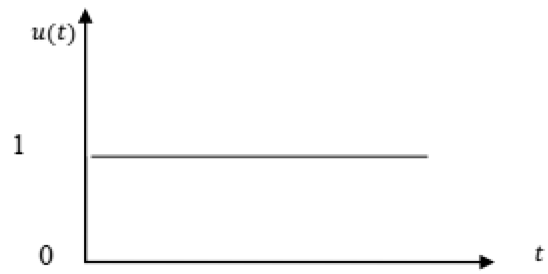


Figure II.1: Unit level

Taking into account the linearity of the Laplace transform, any (non-unitary) step of amplitude A will have the Laplace transform

$$F(p) = TL\big(f(t)\big) = TL(Au(t)) = \frac{A}{p}$$

II.4.2 Ramp or speed step:

This is in reality the integral of the previous function $u(t)$, we generally denote it $v(t)$, such that $v(t)$ = $tu(t)$ (figure II.2)

$$v(t) = \begin{cases} t & t > 0 \\ 0 & t \leq 0 \end{cases}$$

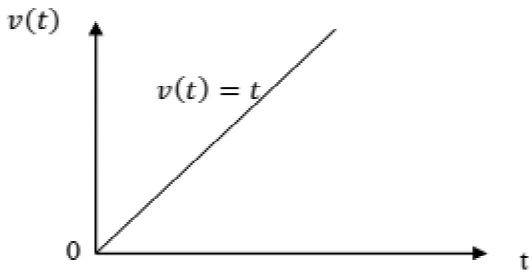


Figure II.2 : Ramp

We obviously have

$$v(P) = TL\big(v(t)\big) = 1/p^2$$

II.4.3 Unit Pulse

By differentiating the function $u(t)$, we obtain a function usually denoted $\delta(t)$, which is called a unit impulse or Dirac impulse.

$$\delta(t) = \frac{du(t)}{dt}$$

It is a zero function everywhere except for $t = 0$, where it has an infinite value.

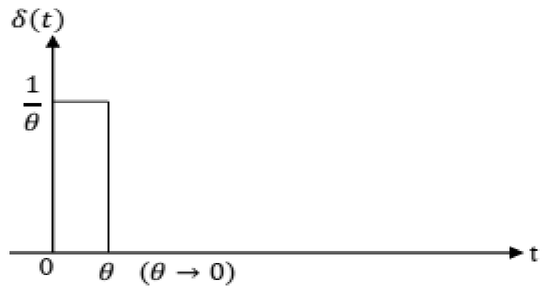


Figure II.3: the Dirac impulse

$\gamma(p) = Tl\big(\delta(t)\big) = 1$

II.4.4 Sinusoidal signal

We consider a signal $s(t) = \sin(wt + \varphi)$ pour $t \geq 0$;

$$s(p) = TL\big(s(t)\big) = \frac{psin\varphi + wcos\varphi}{p^2 + w^2}$$

We will essentially retain the following two results:

For $\begin{cases} s(t) = sinwt \rightarrow s(p) = \frac{w}{p^2+w^2} \\ s(t) = coswt \rightarrow s(p) = \frac{p}{p^2+w^2} \end{cases}$

II.4.5 Table of Laplace Transforms

The table below gives some Laplace transforms. This is only a "mini-table". In engineering sciences, we use much more complete tables.

$f(x)$	$f(t)$, $t \geq 0$
1	$\delta(t)$, unit impulse at t=0
p	$\frac{d}{dt}\delta(t)$, doublet impulse at t=0
$\boldsymbol{\frac{1}{p}}$	$u(t)$, unit step
$\boldsymbol{\frac{1}{p^2}}$	t
$\boldsymbol{\frac{2!}{p^3}}$	t^2
$\boldsymbol{\frac{3!}{p^4}}$	t^3
$\boldsymbol{\frac{m!}{p^{m+1}}}$	t^m
$\boldsymbol{\frac{1}{p+a}}$	e^{-at}
$\boldsymbol{\frac{1}{(p+a)^2}}$	te^{-at}
$\boldsymbol{\frac{1}{(p+a)^3}}$	$\frac{1}{2!}t^2e^{-at}$
$\boldsymbol{\frac{1}{(p+a)^m}}$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
$\boldsymbol{\frac{a}{p(p+a)}}$	$1-e^{-at}$
$\boldsymbol{\frac{a}{p^2(p+a)}}$	$\frac{1}{a}(at-1+e^{-at})$
$\boldsymbol{\frac{b-a}{(p+a)(p+b)}}$	$e^{-at}-e^{-bt}$
$\boldsymbol{\frac{p}{(p+a)^2}}$	$(1-at)e^{-at}$
$\boldsymbol{\frac{a^2}{s(s+a)^2}}$	$1-e^{-at}(1+at)$
$\boldsymbol{\frac{(b-a)p}{(p+a)(p+b)}}$	$be^{-bt}-ae^{-at}$
$\frac{1}{(s-a)^2+\omega^2}$	$\frac{1}{\omega}e^{at}\sin(\omega t)$
$\frac{s-a}{(s-a)^2+\omega^2}$	$e^{at}\cos(\omega t)$
$\boldsymbol{\frac{a}{p^2+a^2}}$	$\sin at$
$\boldsymbol{\frac{p}{p^2+a^2}}$	$\cos at$
$\boldsymbol{\frac{p+a}{(p+a)^2+b^2}}$	$e^{-at}\cos bt$
$\boldsymbol{\frac{b}{(p+a)^2+b^2}}$	$e^{-at}\sin bt$

$\boldsymbol{\frac{a^2+b^2}{p[(p+a)^2+b^2]}}$	$1-e^{at}\left(\cos bt+\frac{a}{b}\sin bt\right)$
$\boldsymbol{\frac{p}{p^2-a^2}}$	$\cosh(at)$
$\boldsymbol{\frac{1}{p^2-a^2}}$	$\frac{1}{\alpha}\sinh(at)$
$\boldsymbol{\arctan(\frac{a}{p})}$	$\frac{1}{t}\sin(at)$
$\boldsymbol{\frac{p^2}{(p^2+a^2)^2}}$	$\frac{1}{2\alpha}(\sinh(at)+\alpha t cos(at))$
$\boldsymbol{\frac{p}{p^2(p+a)}}$	$t-\frac{1}{\alpha}(1-e^{-at})$
$\boldsymbol{\frac{\beta}{(p+a)^2+\beta^2}}$	$e^{-at}\sin(\beta t)$
$\boldsymbol{\frac{p+a}{(p+a)^2+\beta^2}}$	$e^{-at}\cos(\beta t)$
$\boldsymbol{\frac{1}{\sqrt{p}}}$	$\frac{1}{\sqrt{\pi t}}$
$\boldsymbol{\frac{1}{\sqrt{p+a}}}$	$\frac{1}{\sqrt{\pi t}}e^{-at}$
$\boldsymbol{\frac{1}{\sqrt{p^3}}}$	$2\sqrt{\frac{t}{\pi}}$
$\boldsymbol{\sqrt{p-a}-\sqrt{p-\beta}}$	$\frac{1}{2t\sqrt{\pi t}}(e^{-\beta t}-e^{-at})$
$\boldsymbol{\frac{e^{-a\sqrt{p}}}{p}}$	$erfc\left(\frac{a}{2\sqrt{t}}\right)$

II.4.6 Decomposition into simple elements of a rational fraction

We consider the following rational function:

$$F(p) = \frac{\sum_{i=0}^{m} A_i}{\sum_{i=0}^{n} B_i\, P^j} \quad avec\ n \geq m$$

This equation can be put in the form:

$$F(p) = \frac{(p - z_1)(p - z_2) \dots (p - z_i)}{(p - p_1)(p - p_2) \dots (p - p_j)}$$

With m and n the number of roots

z_i: are the roots of F(p) (solution of the numerator).

p_i: are the roots of F(p) (solution of the denominator).

II.5 Exercises

Exercise 1

1- Determine the Laplace transform of the following function:

$$f(t) = 2e^{7t} - 4e^{t}$$

2- Determine the inverse Laplace transform of the following function:

$$F(p) = \frac{(p+3)}{(p+1)(p+2)}$$

Solution :

1) $F(p) = \frac{2}{p+7} - \frac{4}{p+1}$

2) $F(p) = \frac{(p+3)}{(p+1)(p+2)} = \frac{A}{p+1} - \frac{B}{p+2}$ with $p = -1\ et\ p = -2$ denominator solutions,

$F(p) = \frac{2}{p+1} - \frac{1}{p+2}$ => According to the table we have:

$$f(t) = (2e^{t} - e^{t})u(t)$$

Exercise 2

We give $TL(sint) = \frac{2}{p^2+1}$; using the scaling formula

$TL\{(at)\} = \frac{1}{|a|} F\left(\frac{p}{a}\right)$.

1- find $TL[sin(3t)]$.
2- Confirm your result using the Laplace table.

Solution:

$TL(sint) = \frac{2}{p^2+1}$ et $TL(sint) = \frac{2}{p^2+1}$

So $TL[sin(at)] = \frac{1}{a}\frac{1}{\frac{p^2}{a^2}+1} = \frac{1}{a}\frac{a^2}{p^2+a^2}$

If we replace $a = 3$, we find: $TL[sin(3t)] = \frac{3}{p^2+9}$

Exercise 3

Either $F(p) = \frac{p+1}{p(p^2+2p+2)}$

Calculate $f(t)$.

We can write $F(p)$ in the following form: $F(p) = \frac{Ap+B}{(p^2+2p+2)} + \frac{C}{p}$

Solution:

By identification we find : $\begin{cases} A = -\frac{1}{2} \\ B = 0 \\ C = \frac{1}{2} \end{cases}$; So $F(p) = \frac{1}{2}\left(\frac{1}{p} - \frac{p+1}{(p^2+2p+2)}\right)$

According to the Laplace Transform table:

$TL(e^{-at}sinwt) = \frac{w}{(p+a)^2+w^2}$ And $TL(e^{-at}coswt) = \frac{p+a}{(p+a)^2+w^2}$

$$F(p) = \frac{1}{2}\left(\frac{1}{p} - \frac{p+1}{(p^2+2p+2)}\right) = \frac{1}{2}\left(\frac{1}{p} - \frac{p+1}{(p+1)^2+1} + \frac{1}{(p+1)^2+1}\right)$$

So $f(t) = \frac{1}{2}[1 - e^{-t}cost + e^{-t}sint]$.

Exercise 4

Let the differential equation: $\dot{x}(t) + 2x(t) = e^{-t}\ avec\ x(0) = 2$.

Find in $x(t)$ using the T.L.

Solution :

$\dot{x}(t) + 2x(t) = e^{-t}\ \ avec\ x(0) = 2.$

We have $TL[\dot{x}(t) + 2x(t)] = TL[e^{-t}]$

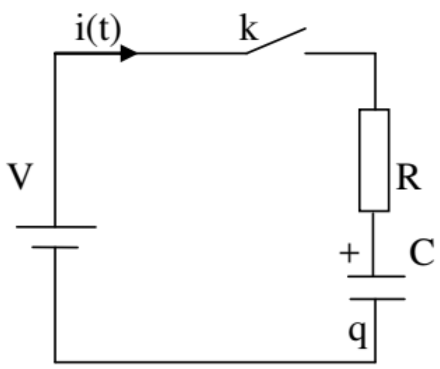
$$px(p) - x(0) + 2x(p) = \frac{1}{p+1}$$

$$x(p)(p+2) = \frac{1}{p+1} + 2$$

$$x(p) = \frac{2p+3}{(p+2)(p+1)} = \frac{1}{p+1} + \frac{1}{p+2}$$

$$TL^{-1}[x(p)] = e^{-2t} + e^{-t}$$

Exercise 5



Consider the following circuit in the figure opposite, V=cte, and in the initial state, C is charged at q.

Write the equations that govern the circuit.

1- Calculate the initial and final values of i(t).
2- Comment on your results.

Solution:

1) $V = Ri(t) + \frac{1}{c}\int i(t)dt$

2) The Laplace transform of this equation is:

$$\frac{V}{p} = Ri(p) + \frac{1}{Cp}[i(p) + q]$$

Such that q is the initial charge of the capacitor, the voltage is constant in the initial state.

$$i(p) = \frac{V - \frac{q}{C}}{p(R + \frac{1}{Cp})} = \frac{V - \frac{q}{C}}{R} \frac{1}{R + \frac{1}{Cp}} => TL^{-1}[i(p)] = \frac{V - \frac{q}{C}}{R} \, e^{\frac{-t}{RC}}$$

$$i(p) = \frac{V - \frac{q}{C}}{R} \frac{1}{R + \frac{1}{Cp}} \; ; i(t) = \frac{V - \frac{q}{C}}{R} \, e^{\frac{-t}{RC}}.$$

The initial value of $i(t)$:$\lim_{t \to 0} i(t) = \lim_{p \to \infty} p.\, i(p) = V - \frac{q}{c}$

The final value of $i(t)$: $\lim_{t \to \infty} i(t) = \lim_{p \to 0} p.\, i(p) = 0$

When contact K closes, the capacity discharges and $i(\infty) = 0$

Chapter III:

Modeling of linear systems

III: Modeling of linear systems

III.1 Introduction

Dynamic systems, whether hydraulic, mechanical, or electrical, can be described by differential equations that are established from the laws of physics which govern the system considered, for example, Kirchhoff's law for electrical systems and Newton's law for mechanical systems. The mathematical description of a given system is called a mathematical model. A dynamic mathematical model describes the behavior of a system over time. In automatic mode a dynamic model is used to:

- The simulation.

- Calculation of controllers.

III.2 Equations of a linear system

A system is said to be linear if the equation linking the output to the input is a linear differential equation with constant coefficients. The general form of this differential equation is:

$$b_0 s(t) + b_1 \frac{ds(t)}{dt} + \cdots + b_n \frac{d^n s(t)}{dt^n} = a_0 e(t) + a_1 \frac{de(t)}{dt} + \cdots + a_m \frac{d^m e(t)}{dt^m} \qquad \text{(III. 1)}$$

These linear systems are homogeneous, that is to say $s(k.e) = k.s(e)$, and additive, that is to say that we have $s(e1 + e2) = s(e1) + s(e2)$.

We call the order of equation (III.1), (n) the order of the linear system. Only systems for which m ≤ n are encountered in practice.

III.3 Examples of modeling

III.3.1. Electrical first order system (RC Circuit)

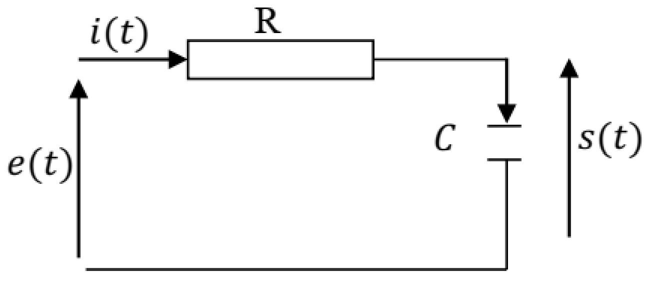


Figure III.1: RC Circuit

We consider the RC circuit in Figure III.1. We consider that it is a system having an input e(t) and an output $s(t)$; we can apply Kirchhoff's second law as well as Faraday's law.

$$e(t) = Ri(t) + s(t) \qquad \text{(III. 2)}$$

$$i(t) = c\frac{ds(t)}{dt} \qquad \text{(III. 3)}$$

Replacing (III.3) in (III.2) we obtain the differential equation

$$Rc\frac{ds(t)}{dt} + s(t) = e(t) \qquad \text{(III. 4)}$$

This system is therefore modeled by a first-order differential equation. The system is also said to be top-notch.

III.3.2. Electrical second order system (RLC circuit)

We now consider the system presenting an RLC circuit as shown (figure III.2).

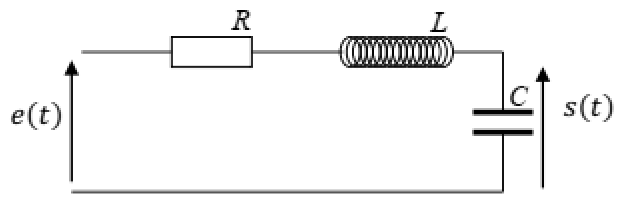


Figure III.2: RLC Circuit

We can write with the same principle as for the previous example.

$$e(t) + L\frac{di(t)}{dt} + Ri(t) + s(t) = 0 \qquad \text{(III. 5)}$$

$$i(t) = c\frac{ds(t)}{dt} \qquad \text{(III. 6)}$$

As before, we can obtain a differential equation connecting the output $s(t)$ to the input $e(t)$:

$$LC\frac{d^2s(t)}{dt^2} + Rc\frac{ds(t)}{dt} + s(t) = e(t) \qquad \text{(III. 7)}$$

The equation thus found is a second-order differential equation that models a second-order electrical system.

III.3.3 Mechanical second order system

We consider the mechanical system of Figure III.3, where a mass is subjected to a force $e(t)$, a restoring force from the spring $\overrightarrow{f_k} = -k\vec{x}(t)$ and a viscous friction force $\overrightarrow{f_k} = -f\,\frac{d\vec{x}}{dt}$

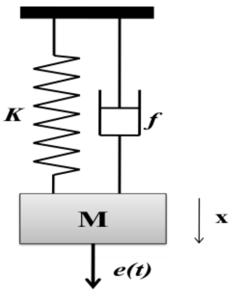


Figure III.3: Example of a mechanical second order system

The fundamental relation of dynamics gives us:

$$M\frac{d^2x(t)}{dt^2} = e(t) - f\frac{dx(t)}{dt} - kx(t) \qquad \text{(III. 8)}$$

$$M\frac{d^2x(t)}{dt^2} + f\frac{dx(t)}{dt} + kx(t) = e(t) \qquad \text{(III. 9)}$$

III.3.4. Electromechanical system (Electric motor)

This is the modeling of a system for controlling the speed of a DC motor. This model can represent that of a motor that controls a joint of a robot arm. It is very common in enslavements. Its diagram is as follows:

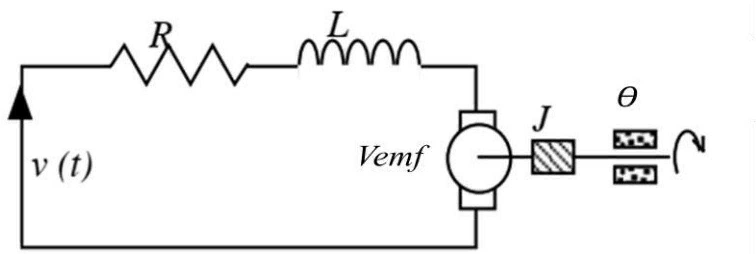


Figure III.4: Electric motor diagram

The motor is powered by a voltage *V*. The motor with its winding is equivalent to an electrical circuit of resistance *R*, inductance *L* and a counter-electromotive force V_{emf}. The motor operates a mechanical system of moment of inertia J, including the rotor and the load. Presence of friction of coefficient *b*. The mechanical system rotates through an angle θ.

We consider that the constants are such that Kt (constant characterizing the armature) is equal to Ke (motor constant): $K=K_e=K_t$.

The moment applied by the motor torque, T, is proportional to the armature current i, by a constant factor K_t. The counter-electromotive force (V_{emf}) is linked to the rotation speed by the following equations:

$$T = K_t . i \tag{III.10}$$

$$e(t) = K_e \frac{d\theta(t)}{dt} \tag{III.11}$$

By applying Newton's and Kirchhoff's laws, we obtain the following equations:

$$j\ddot{\theta} + b\dot{\theta} = K.i \tag{III.12}$$

$$Rc\frac{di}{dt} + Ri = V - K\dot{\theta} \tag{III.13}$$

III.3.5. Hydraulic system

As for the example of the electrical circuit, a hydraulic system consists of a tank that is supplied by a liquid inlet pipe and that delivers a certain flow rate of liquid at the output. The tank has a surface A, and the liquid level is marked by the height x. The flow rate of the pipe that brings the liquid is noted. V_{in}, the flow rate of the liquid leaving the tank, is noted as V_{out}.

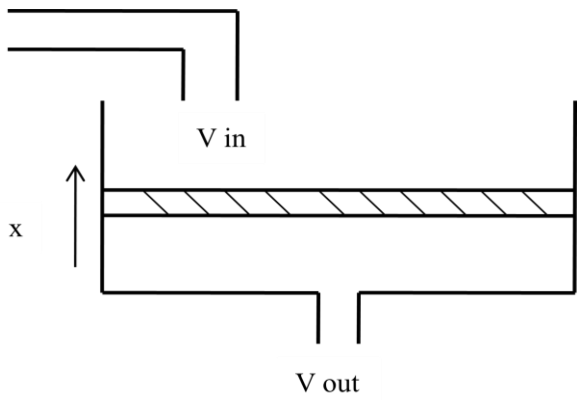


Figure III.5: model of a hydraulic system

a- Determine the entrance and exit: The quantity that we wish to control in the context of this problem is the height of the liquid level in the tank. This is therefore the output quantity. The input is the V_{in} flow of the liquid supplying the tank. We can even define the input as the difference between the incoming flow and the outgoing flow: (V_{in} –V_{out}).

b- Find the link between output and input. The mathematical model is obtained using the principle of conservation of matter. The volume variation in the tank is given by the relation:

$$A\frac{dx}{dt} = V_{in} - V_{out} \qquad \text{(III. 14)}$$

III.3.4. Remarks

III.3.4.1 Static regime

In equation (III.1), if the successive derivatives of the input e(t) and the output s(t) are zero, we obtain $b_0 s(t) = a_0 e(t)$. We define the static gain K of the system as being the ratio

$$K = \frac{a_0}{b_0}$$

III.3.4.2 Initial conditions

In the rest of the course, we will often assume that the initial values of the input and output are zero. In fact, if this is not the case but we find ourselves in rest conditions of the system, we can show that the variations around this equilibrium point verify the same equation (III.1) as the quantities themselves.

III.3.4.3 Linearization

Real systems are sometimes not linear but can be considered as such under certain conditions. In the remainder of the course, we will only study linear or linearizable systems.

III.3.4.4 Response of a linear system

If we want to know the answer of a linear system, we just need to solve equation (III.1). In the rest of the course, we will use the Laplace Transform (TL) to simplify the resolution of these equations. We will also learn to make a direct link between the responses of the systems and the TL of equation (III.1).

III.4. Concept of Transfer Function

III.4.1 Definition

The transfer function, or transmittance, of a linear system is called the ratio between the Laplace transform of the output and that of the input:

$$T(P) = \frac{S(p)}{E(p)} = \frac{a_0 + a_1.p + \cdots + a_m.p^m}{b_0 + b_1.p + \cdots + b_n.p^n} \qquad \text{(III. 15)}$$

It is a rational function. The order of the system (which is the order of the differential equation) is the degree of the denominator of T (p). To express the previous equation, we generally use the diagram in figure III.6.

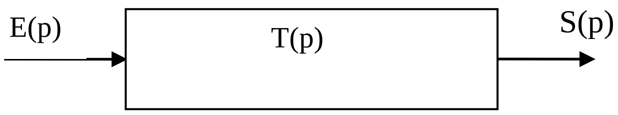


Figure III.6: Functional diagram of a transfer function

III.4.2 Different forms of writing the transfer function

We have previously seen the expanded form of the transfer function, where we can directly read the coefficients of the differential equation.

$$T(P) = \frac{S(p)}{E(p)} = \frac{a_0 + a_1.p + \cdots + a_m.p^m}{b_0 + b_1.p + \cdots + b_n.p^n} \qquad \text{(III. 16)}$$

It is often preferable to highlight the gain K of the system as well as the number α of pure integrators, also called the type of the system.

$$T(P) = K.\frac{1}{p^\alpha}\frac{1 + \cdots + c_m p^m}{1 + \cdots + d_{n-\alpha}p^{n-\alpha}} = K.G(p) \qquad \text{(III. 17)}$$

Noticed

If $\alpha = 0$, So $K = \frac{a_0}{b_0}$ is the static gain of the system.

If $\alpha \neq 0$, So $K = \lim_{p\to 0} p^\alpha T(p)$.

This last form can sometimes be found in factorized form:

$$T(P) = K.\frac{(1+\tau_1' p) \dots (1+\tau_m' p)}{p^\alpha(1+\tau_1' p) \dots (1+\tau_n' p)}$$

In this formulation, the τ and τ_0 are assimilated to time constants.

We can finally show the poles and zeros of the transfer function. This gives:

$$T(P) = k.\frac{(p - z_1) \dots (p - z_m)}{p^\alpha (p - p_1) \dots (p - p_{n-\alpha})}$$

$k \neq K$

III.4.3 Transfer function example (electrical system)

Let us consider the following (simple) electrical system, where we will define the input u(t) as the supply voltage and the output i(t) as the current flowing in the circuit.

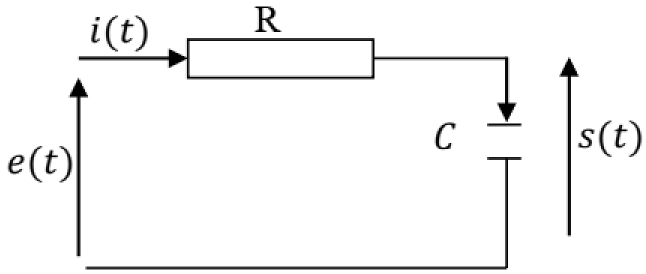


Figure III.7: RC Circuit

$$Rc\frac{ds(t)}{dt} + s(t) = e(t)$$

Applying the Laplace transformation to both sides of this equation while assuming the different initial conditions to be zero

$Rc\frac{ds(t)}{dt} + s(t) = e(t)$

$RCpS(p) + S(p) = E(p)$

$(RCp + 1)s(p) = E(p)$

$=> \frac{S(p)}{E(p)} = \frac{1}{(1 + RCp)}$

We can form the transfer function of this system

$$T(p) = \frac{s(p)}{E(p)} = \frac{1}{(1 + RCp)}$$

$$T(p) = \frac{1}{(1 + RCp)}$$

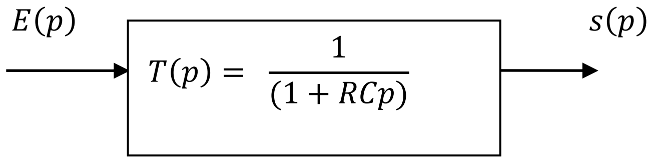


Figure III.8: Functional diagram of an RC Circuit

We will easily identify the fact that it is a system of order 1 whose time constant is τ = RC and static gain K = 1.

III.5. Functional diagram

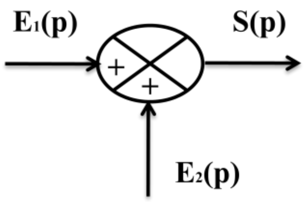
III.5.1 Definition

A block diagram is a simplified representation of a transfer function of a process. In other words, it is a graphic that can involve elementary symbols such as summator, comparator, sensor, etc.

There are four basic diagrams used in the functional representation of controlled systems:

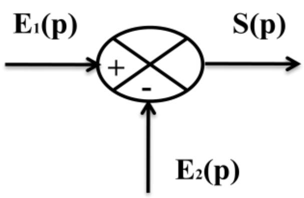
- **Block** S(p)=H(p).E(p)

- **Summation**



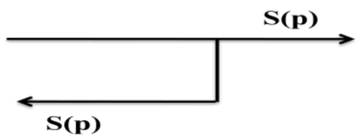
S(p)=E_1(p)+E_2(p)

- **Comparator**



S(p)=E_1(p)-E_2(p)

- **Sensor**



III.5.2 Representation of an Open Loop Transfer Function (FTBO):

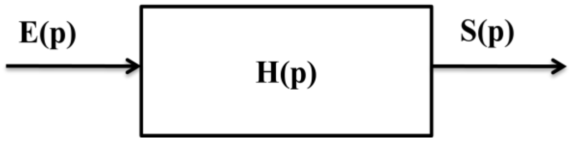


Figure. III.9*:* Functional diagram of an FTBO

$$S(p) = H(p).E(p) \;=>\; H(p) = \frac{S(p)}{E(p)}$$

$H(p)$: Open Loop Transfer Function

III.5.3 Representation of a Closed Loop Transfer Function (FTBF):

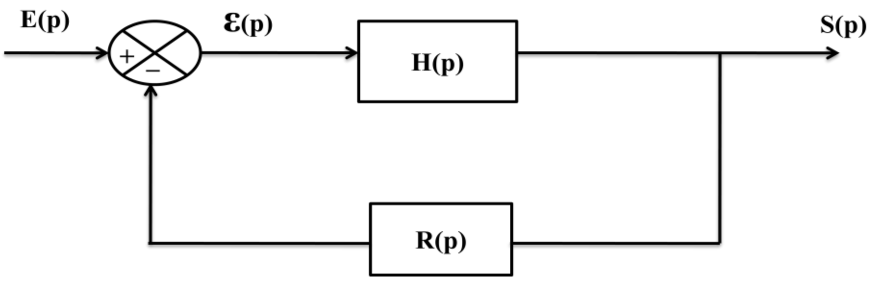


Figure III.10: Functional diagram of an FTBF

$S(p) = H(p).\varepsilon(p)$, avec $\varepsilon(p) = E(p) + R(p).S(p)$

$S(p) = H(p).\big(E(p) - R(p).S(p)\big) = H(p).E(p) - H(p)R(p).S(p)$

$S(p) + H(p)R(p).S(p) = H(p).E(p)$

$S(p)\big(1 + \; H(p)R(p)\big) = H(p).E(p)$

The Closed Loop Transfer function:

$$=> \frac{S(p)}{E(p)} = \frac{H(p)}{\left(1 + H(p)R(p)\right)}$$

For a unitary return system $R(p) = 1$, the FTBF become:

$$\frac{S(p)}{E(p)} = \frac{H(p)}{\left(1 + H(p)\right)}$$

III.5.4 Simplification of the functional diagram:

Simplifying a diagram or reducing it amounts to making transformations to highlight the transfer function. To do this, we will use a certain number of elementary rules.

III.5.4.1 Association of several elements in cascade (series):

If we mount n elements of transfer functions H_1, H2, … Hn in cascade (in series), they are equivalent to a single element H whose transfer function is given by: H= H_1,H_2, … Hn.

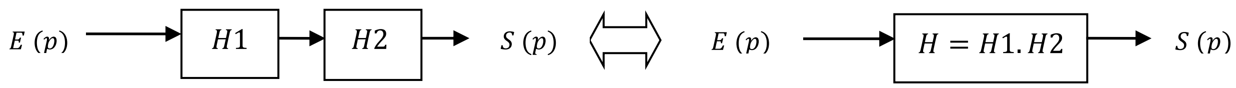


Figure III.11: Cascading Blocks

III.5.4.2 Blocks in parallel:

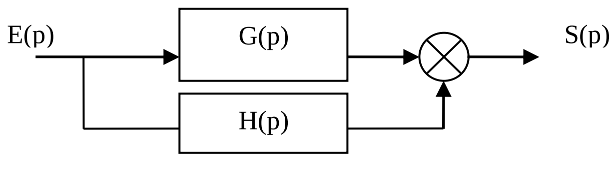


Figure III.12: Parallel blocks

$$S(p) = E(p)G(p) + E(p)H(p) = E(p)(G(p) + H(p))$$

$$\frac{S(p)}{E(p)} = G(p) + H(p) = T(p)$$

III.5.4.3 Closed loop systems:

It is a system whose control signal depends on the output.

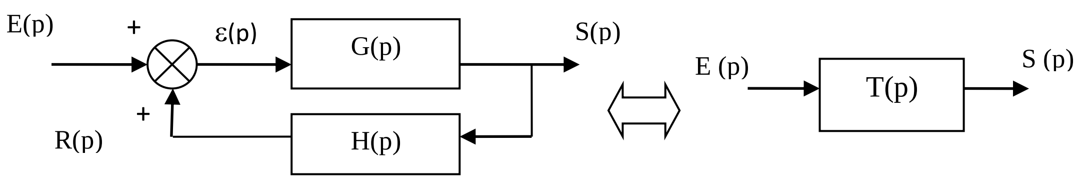


Figure III.13: Representation of a closed loop

According to the functional diagram we can write:

$S(p) = \varepsilon(p) G(p)$

$R(p) = S(p) H(p)$

$\varepsilon(p) = E(p) + R(p).$

$S(p) = [E(p) + R(p)] G(p) \Rightarrow S(p) = [E(p) + S(p) H(p)] G(p)$

$\Rightarrow S(p) = E(p) G(p) + S(p) H(p) G(p) \Rightarrow S(p)(1 - H(p) G(p)) = E(p) G(p)$

$$\Rightarrow \frac{S(p)}{E(p)} = \frac{G(p)}{1 - H(p)G(p)} = T(p)$$

III.5.4.4 Closed loop systems with unitary feedback:

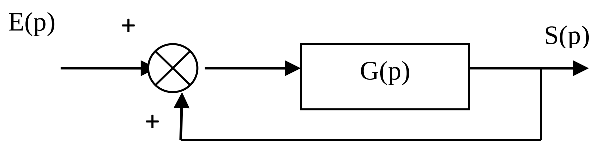


Figure III.14: Representation of a closed loop with unitary feedback

$$\frac{S(p)}{E(p)} = \frac{G(p)}{1 - G(p)} = T(p)$$

III.5.4.5 Association of two comparators:

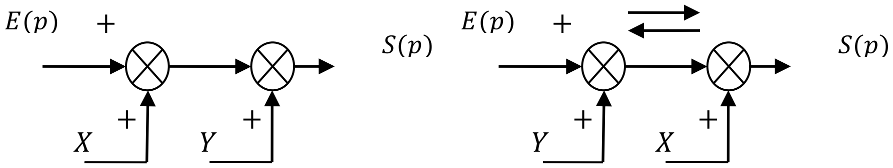


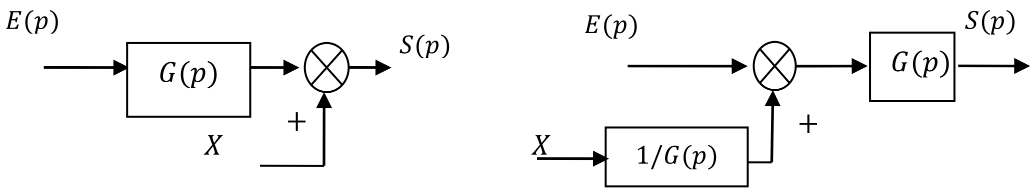
Figure III.15: Permutation of adders

$S(p) = E(p) + X + Y = E(p) + Y + X$

III.5.4.6 Moving comparators:

III.5.4.6.1 To the left of an element (Moving a downstream block upstream):

This type of movement results in the addition of a functional block with a transfer function equal to the input of the adder, which takes the place of the comparator as shown in Figure III.16.



$S(p) = E(p)G(p) + X$ $\qquad$ $S(p) = \left(E(p) + \frac{X}{G(p)}\right) G(p)$

Figure III.16: Moving a downstream block upstream

$$S(p) = E(p)G(p) + X = \left(E(p) + \frac{X}{G(p)}\right) G(p)$$

III.5.4.6.2 To the right of an element (Moving a block from upstream to downstream):

By reasoning similar to the previous case, we obtain:

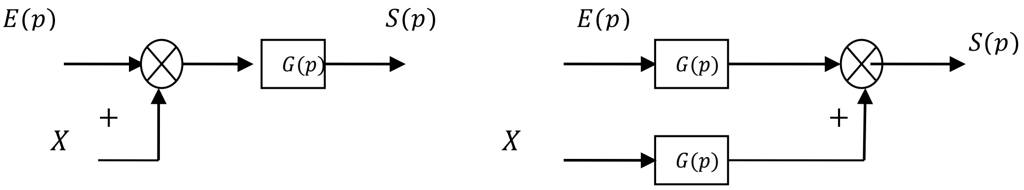
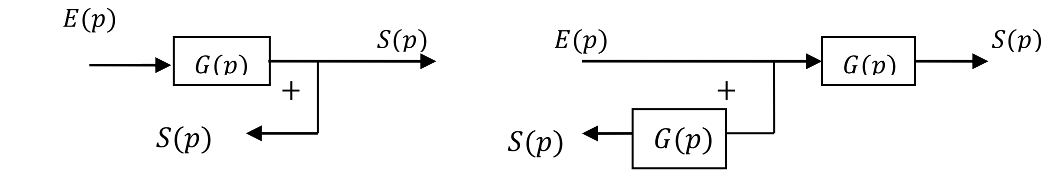


Figure III.17: Moving a block from upstream to downstream

$S(p) = G(p)(E(p) + X) = (G(p)E(P) + G(p)X)$

III.5.4.7 Moving a branch point: (Moving a sensor)

III.5.4.7.1 To the left of an element



$S(p) = E(p)G(p)$ $\qquad\qquad$ $S(p) = E(p)G(p)$

Figure III.18: Moving a downstream block upstream

III.5.4.7.1 To the right of an element:

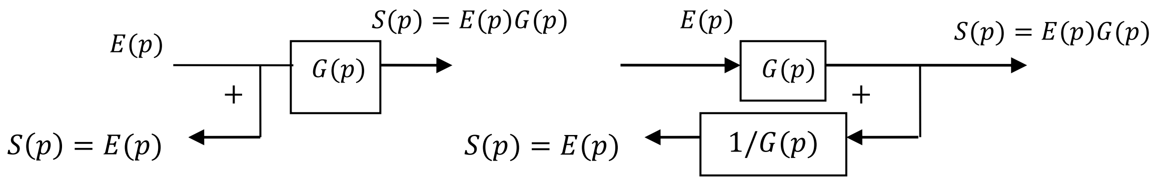


Figure III.19: Moving a downstream block upstream

III.5.5 Multiple input systems. Application of the principle of superposition:

Some systems are described by a block diagram comprising several inputs and/or several outputs. Giving the transfer functions of such a system consists of writing each of the outputs as a function of all the inputs. To calculate these transfer functions, the method is to use the principle of superposition of linear systems: for each input signal, we calculate each of the outputs without considering the other inputs (we act as if they were zero). We then sum the transfer functions thus found for each output. When we have several signals in a linear system, we treat each of them independently of the others. The output signal produced by all signals acting at the same time is calculated as follows:

1. Make all signals null, except one.

2. Calculate the response produced by the chosen signal acting alone.

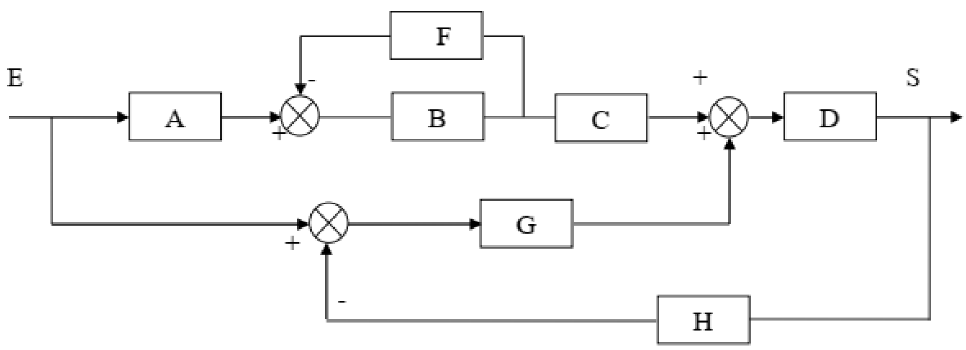
3. Repeat steps 1 and 2 for each of the remaining input signals.

4. Algebraically add all the calculated answers. This sum represents the total output magnitude obtained when all input signals act together.

III.6 Examples

Example 1

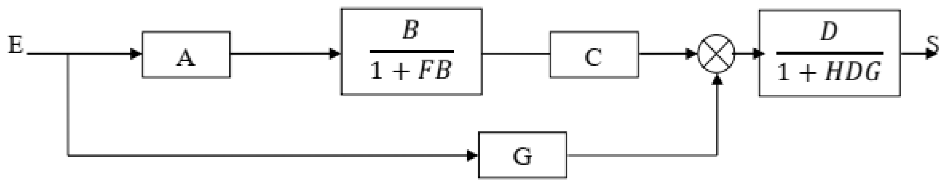
Consider the following block diagram:



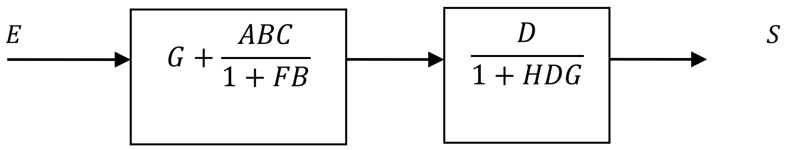
- Calculate its transfer function.

Solution :

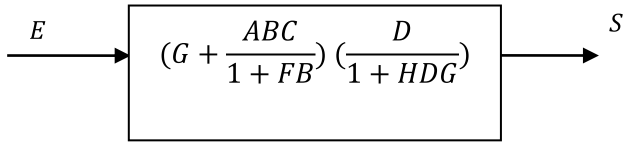
- **a)**



- **b)**



- **c)** Finally: cascading

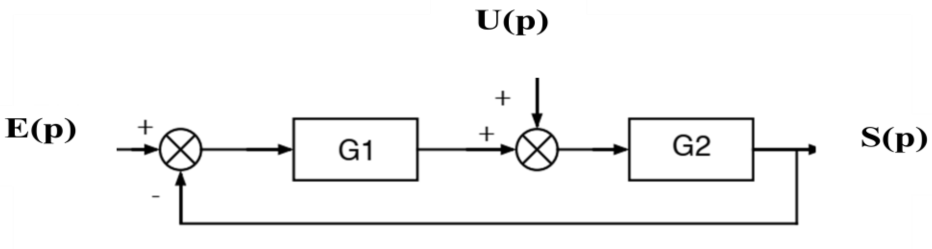


So the transfer function:

$$\frac{S}{E} = \frac{DG + FBDG + ABCD}{(1 + FB)(1 + HDG}$$

Example 2

In the system described in the figure below, we notice two inputs E and U, and an output S.



Let's calculate S as a function of U (on pose $E = 0$):

$$S_u(p) = \frac{G_2}{1 + G_1 G_2} U(p)$$

Let us calculate S as a function of E (we set U = 0):

$$S_e(p) = \frac{G_1 G_2}{1 + G_1 G_2} E(p)$$

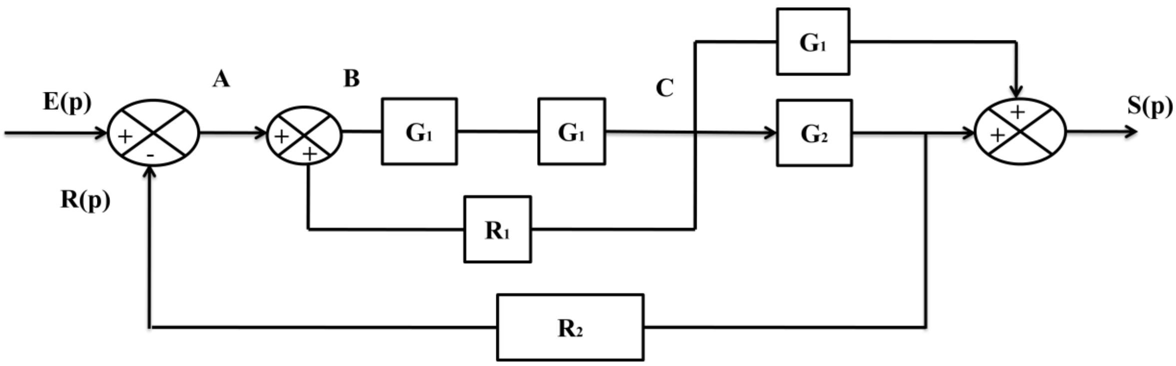
Which give :

$$S(p) = \frac{G_2}{1 + G_1 G_2} U(p) + \frac{G_1 G_2}{1 + G_1 G_2} E(p)$$

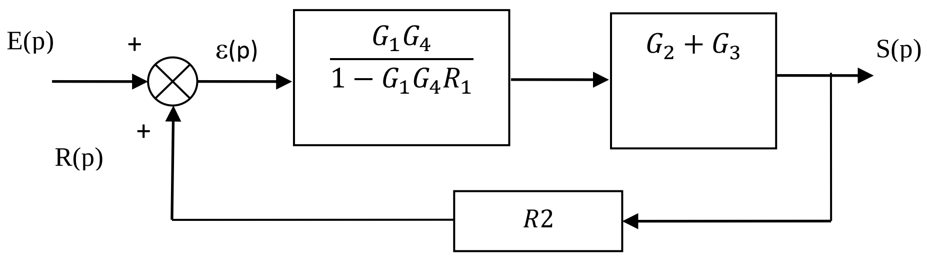
III.6 Exercises

Exercise 1

A system is described in the figure below; the G_i and the R_i are transfer functions. We look for the transfer function equivalent to the set.



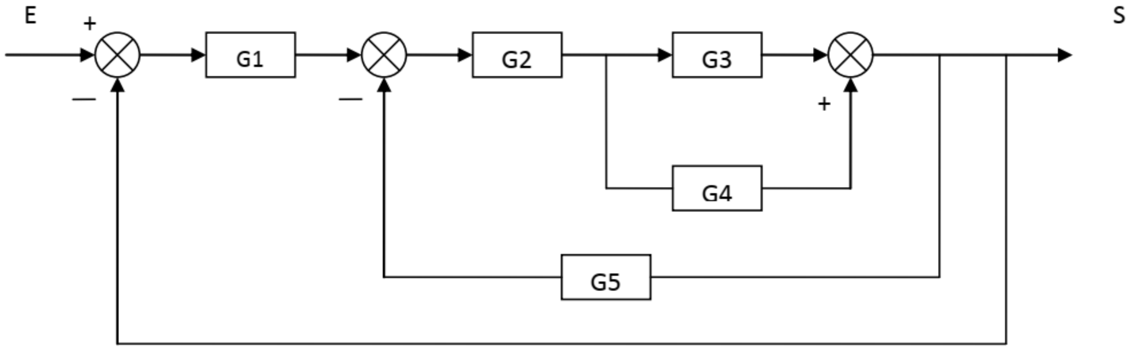
Block diagram after simplifications:



$$S = \frac{(G_2 + G_3) G_1 G_4}{1 - R_1 G_1 G_4 + (G_2 + G_3) G_1 G_4 R_2} E$$

Exercise 2

Find the equivalent transfer function for the following slave system:



Calculate its transfer function.

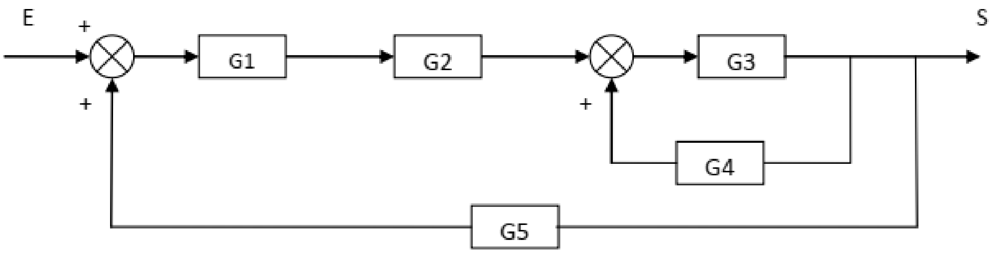
Solution:

The transfer function H(p) is:

$$H(P) = \frac{G_1(G_2G_4 + G_2G_3)}{1 + G_2G_5(G_4 + G_3) + G_2G_1(G_3 + G_5)}$$

Exercise 3

Consider the following block diagram:



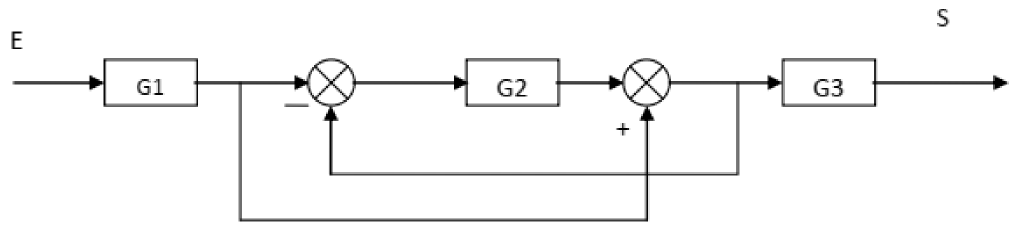
Calculate its transfer function $T(p)$.

Solution: The transfer function T(p) is:

$$T(P) = \frac{G_1G_2G_3}{1 + G_2G_3 + G_2G_1}$$

Exercise 4

Find the output signal S for the following servo system:

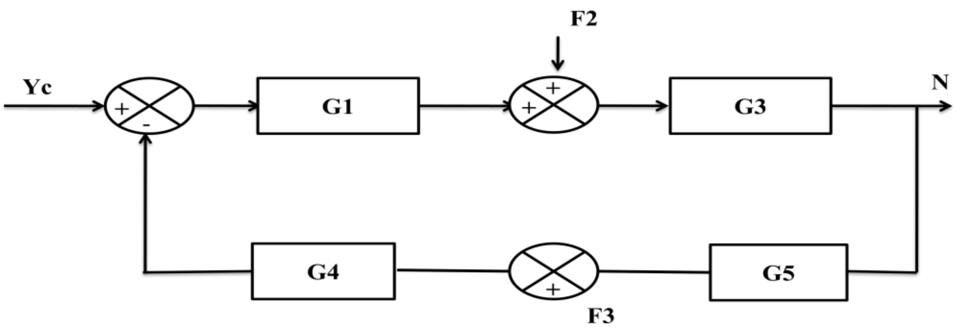


Solution:

$T(P) = G_1G_3$

Exercise 5

The figure below represents a functional diagram of a servo system:



Find the output signal S for the slave system.

Solution:

Application of the principle of superposition:

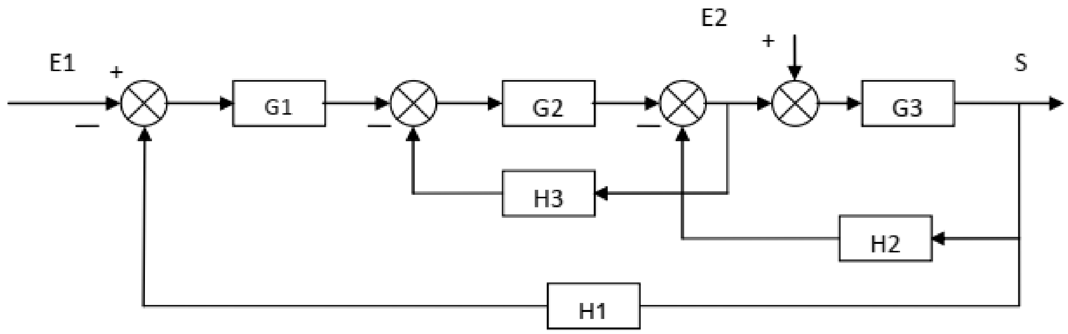
$$E_2 = E_3 = 0 \ => S_1 = E_1 \frac{G_1G_2}{1 + G_1G_2G_3G_4}$$

$$E_1 = E_3 = 0 \ => S_2 = E_2 \frac{G_2}{1 + G_1G_2G_3G_4}$$

$$E_1 = E_2 = 0 \implies S_3 = E_3 \frac{G_1 G_2 G_3}{1 + G_1 G_2 G_3 G_4}$$

Exercise 6

Consider the following block diagram:



Simplify the block diagram and find the equivalent transfer function.

Solution:

Application of the principle of superposition:

$$E_2 = 0 \implies S_1 = E_1 \frac{G_1 G_2 G_3}{1 + G_1 H_2 + G_2 H_3 + G_1 G_2 G_3 H_3}$$

$$E_1 = 0 \implies S_2 = E_2 \frac{G_3 + G_1 H_3}{1 + G_1 H_2 + G_2 H_3 + G_1 G_2 G_3 H_3}$$

Exercise 7

We apply a pulse to the input of a controlled system, and we observe the function e^{-2t} for the output signal.

Find the system transfer function

Solution: The system Transfer function:

$$H(p) = \frac{1}{p + 2}$$

Exercise 8

The output $s_1(t)$ of a slave system is:$2(1-\ e^{-2t})$ for an input $e_1(t)=2$.

Find the output of this system for an input $e_2(t)=2t$

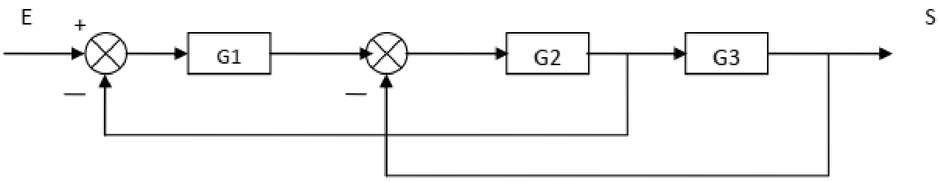
Solution:

$H(p)=\frac{1}{p^2(p+2)}$, After decomposition we find: $s(t)=-1+2t+e^{-2t}$

III.7 Additional exercises

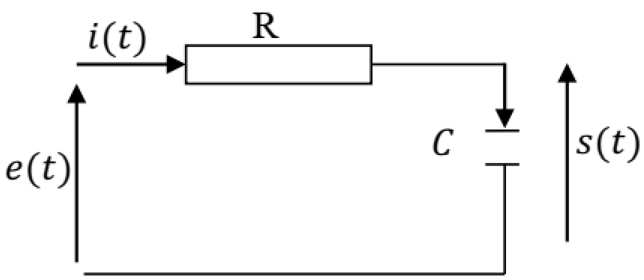
Exercise 1

Simplify the block diagram and find the equivalent transfer function for the following servo system:



Exercise 2

We consider the following (simple) electrical system, where we define the input u(t) as the supply voltage and the output i(t) as the current flowing in the circuit.



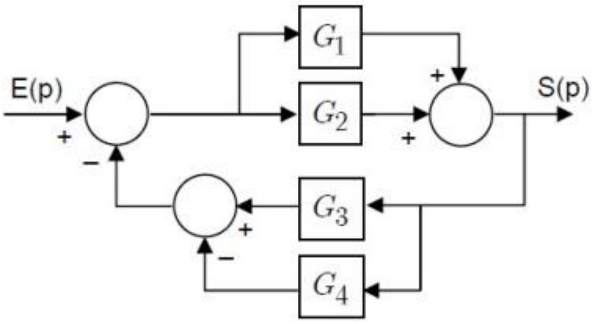
1- Determine the transfer function T(P).

2- Determine expressions for time constant τ and static gain K.

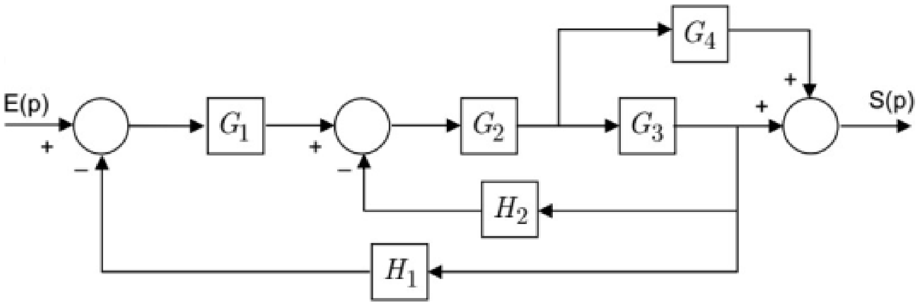
Exercise 3

Determine the transfer functions by simplifications of the functional blocks:

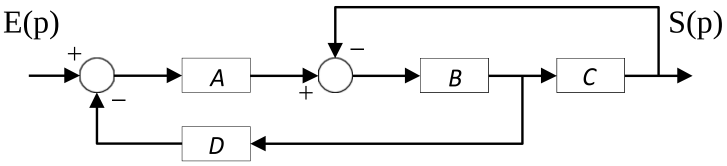
a)



b)



c)



Chapter IV :

Temporal response of linear systems

IV. Temporal response of linear systems

IV.1 Introduction

Temporal analysis consists of studying the response of a system represented by its transfer function to a time-varying input signal. The input signal can in principle be any. However, to obtain an analytical expression, we will use elementary signals (pulse, step, ramp).

This is justified by the fact that any signal can be decomposed into a sum of elementary signals. Typically, much can be learned about systems by observing the response to the following inputs:

- Dirac impulse: Impulse response

- Echelon: Index response

- Ramp: Speed response

- Sinusoid: Frequency response

In the following chapter we will study the frequency responses of the systems. In this chapter, we will make the link between transfer function and temporal responses (i.e. responses to pulses, step and ramp). As in the rest of the course, we will study the simple and very widespread systems that are the first order and second order systems. In addition, the methods of studying these systems are easily generalized to others.

IV.2 first order linear systems

IV.2.1 Definition

A first order system is any system whose operation is described by a first order differential equation.

$$\tau \frac{ds(t)}{dt} + s(t) = e(t) \Rightarrow \tau p\, S(p) + S(p) = E(p) \Rightarrow T(p) = \frac{S(p)}{E(p)} = \frac{1}{1+\tau p}$$

$T(p)$: Transfer function of the first order system.

IV.2.2 Index responses of a first-order system

We call the index response of a function the response s(t) to a known and non-periodic input e(t). Entries giving index answers:

IV.2.2.1 Step input

This is the most used entry of all. It corresponds to a sudden change of setpoint.

This function is defined by: $e(t) = \begin{cases} 0 & \forall t \leq 0 \\ a & \forall t \rangle 0 \end{cases}$

Its Laplace transform is: $E(p) = TL[e(t)] \ = \frac{a}{p}$

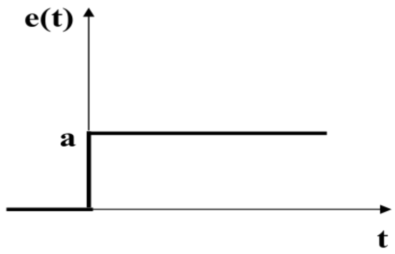


Figure IV.1: The step function

We call unit level the function whose TL is $\frac{1}{p}$ (a = 1). It is often denoted $u(t)$. we call the index response the response at the unit level.

IV.2.2.2 Ramp input

The ramp of slope a is the antiderivative of the height step a. It is defined by:

$$e(t) = \begin{cases} 0 & t \langle 0 \\ at & t \rangle 0^{+} \end{cases}$$

Its Laplace transform is defined by: $E(p) = TL[e(t)] = \frac{a^2}{p}$

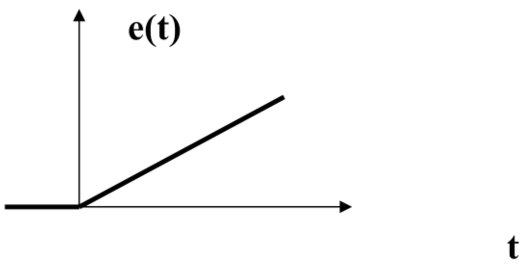


Figure IV.2: The slope ramp function has a

We can also define the unit ramp: the ramp of slope 1.

IV.2.2.3 Pulse input

The unit impulse is, in the space of distributions, the derivative of the unit scale. It is also called the Dirac impulse. It is generally denoted by $\delta(t)$.

It is defined by:

$$e(t) = \begin{cases} 0 \ \forall \ t \neq 0 \\ \delta t \ \forall \ t = 0 \end{cases}$$

Its Laplace transform is $TL[\delta(t)] = 1$.

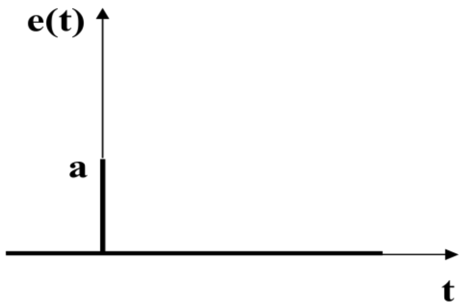


Figure IV.3: The Dirac impulse function of weight a

IV.2.2.4 Sinusoidal input (Sinusoidal signal)

This signal is the basic signal for the frequency study of linear systems, i.e., the frequency response of the system: $e(t) = Asin(wt)u(t)$

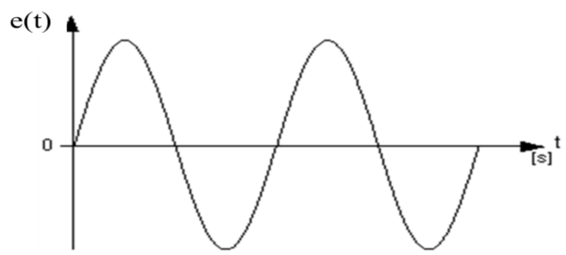


Figure IV.4: sinusoidal

IV.3 Response of a first order system

IV.3.1 Transfer function

A first order system is described by

$$b_0 s(t) + b_1 \frac{ds(t)}{dt} = a_0 s(t) + a_1 \frac{de(t)}{dt}$$

In this chapter, we will only deal with systems for which a $a_0 \neq 0$ and $a_1 \neq 0$.

The transfer function of these systems is:

$$T(p) = \frac{a_0}{b_0 + b_1 p}$$

Which we can put in the form

$$T(p) = \frac{K}{1 + \tau p}$$

We call K the static gain and τ the time constant of the system.

IV.3.2 Response at one level

For all the individual responses (at one level) we define:

- Steady state $S_p(t) = S(t) \quad \forall\, t \gg t_r \quad (S_p(t) = \lim_{t \to \infty} s(t))$
- Rise time t_m is the time during which s(t) passes from 0.1 $s_p(t)$ has 0.9 $s_p(t)$.
- Response time at 5% t_r is the time after which: $\forall\, t > t_r, s_p(t) - s(t) < 0.05 s_p(t)$.

An amplitude E_0 is applied to the input of this system.

$E(p)$, The TL of the entrance is therefore $E(p) = \frac{E_0}{p}$.

The output of the system is such that: $s(p) = E(p)T(p) = \frac{KE_0}{p(1+\tau p)}$

Applying the inverse Laplace Transform we find $s(t) = KE_0(1 - e^{-\frac{t}{\tau}})$

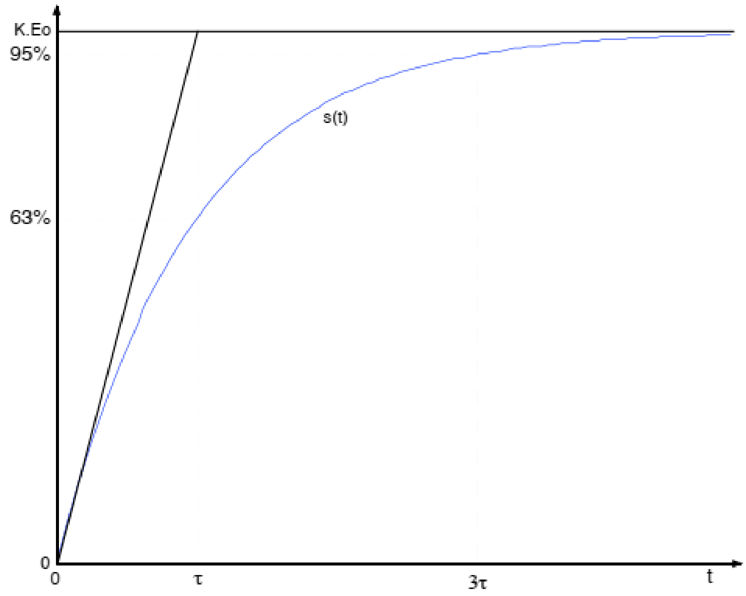


Figure IV.5: Response at a level of a first order system

According to Figure IV.5, we can see:

- $s(\tau) = 0.63\, KE_0$
- $\lim_{t\to\infty} s(t) = KE_0$
- La tangente à l'origine a une pente de $\frac{KE_0}{\tau}$
- Temps de montée $T_m \approx 2\tau$
- Temps de réponse à 5% $T_r \approx 3\tau$

We can trace the curve in reduced coordinates, that is to say, the trace of $y = \frac{s(t)}{KE_0}$ as a function of $y = \frac{s(t)}{KE_0}$ which no longer depends on τ nor on K nor on the input amplitude $y = 1 - e^{-x}$

IV.3.3 Response to a ramp

The input is a ramp with slope a: $e(t) = atu(t)$. Its Laplace transform is $E(p) = \frac{a}{p^2}$

The output is given by $s(p) = \frac{K.a}{\tau}\frac{1}{p^2(p+\frac{1}{\tau})}$

$$s(t) = K.a.(t-\tau) + K.a.\tau.e^{-\frac{t}{\tau}}$$

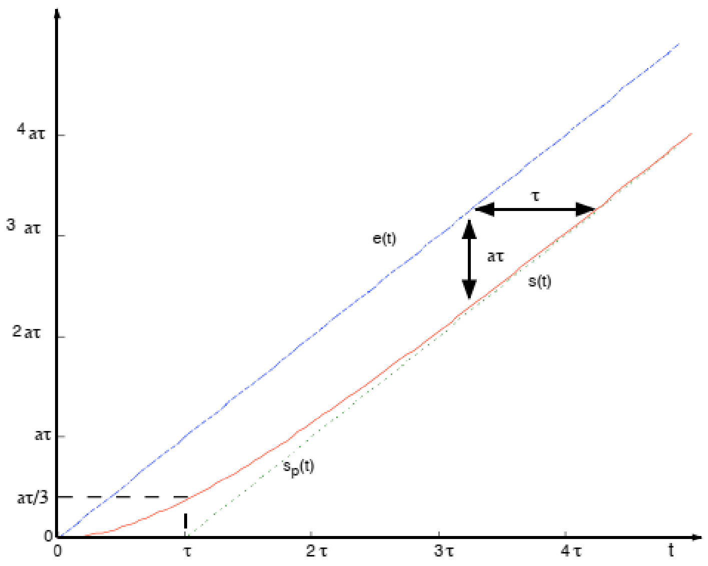


Figure IV.6 : First order response to a ramp

The characteristics of this response are:

- The steady state is $Sp(t) = K.a.(t-\tau)$
- If $K = 1$ the output $s(t)$ follows the input with a constant delay (τ) the difference between the output and the input is called trailing error and is equal to $a.\tau$.
- If $K \neq 1$, $sp(t)$ and $e(t)$ do not have the same slope. They diverge.

IV.3.4 Response to an impulse

The entry is given by $e(t) = E_0\delta(t)$. With $E(p) = E_0$.

The output is given by $Sp(t) = \frac{KE_0}{(1+\tau p)}$ => $s(t) = \frac{KE_0}{\tau}$

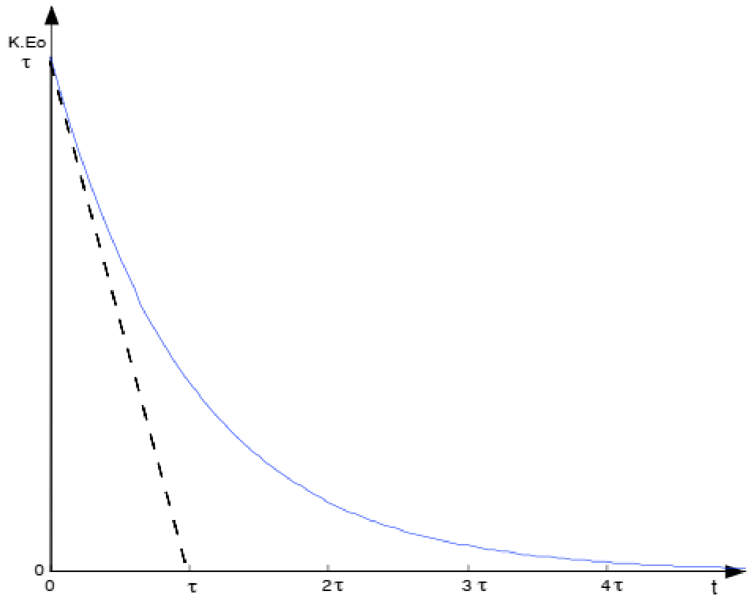


Figure IV.7: Response of a first order system to an impulse

IV.4 Systèmes linéaires du deuxième ordre

IV.4.1 Definition:

A second order linear system is described by a second order differential equation:

$$T^2 \frac{d^2 s(t)}{dt^2} + 2\zeta T \frac{ds(t)}{dt} + s(t) = K_s e(t)$$

This equation can be put in the following form:

$$T^2 p^2 S(p) + 2\zeta TpS(p) + S(p) = K_s E(p)$$

We can therefore write that the transfer function of the system is:

$T(p) = \dfrac{S(p)}{E(p)} = \dfrac{K_s}{T^2 p^2 + 2\zeta Tp + 1}$ avec $\zeta>0$, and T>0.

We pose: $T=1/\omega_n$

Where: $T(p) = \dfrac{S(p)}{E(p)} = \dfrac{1}{\dfrac{1}{\omega_n^{\,2}} p^2 + \dfrac{2\zeta\omega_n}{\omega_n^{\,2}} p + 1} = \dfrac{K_s \omega_n^{\,2}}{p^2 + 2\zeta\omega_n p + \omega_n^2}$

K_s : Static gain.

ω_n: Natural frequency of the undamped system.

ζ: The damping ratio or damping coefficient.

$\alpha = \zeta/T = \zeta\omega_n$: The damping coefficient.

$1/\alpha = 1/\zeta\omega_n$: The time constant.

The transfer function $T(p)$ can have two roots:

$$T^2p^2 + 2\zeta Tp + 1 \Rightarrow (p - p_1)(p - p_2) = (p - \frac{\zeta + \sqrt{\zeta^2 - 1}}{T})(p - \frac{\zeta - \sqrt{\zeta^2 - 1}}{T})$$

- $\zeta>1 \Rightarrow p_1$ and p_2 are two negative or positive real roots ⇒ the system is stable. (Revving on cushioning).
- $\zeta=1 \Rightarrow p_1$ and p_2 are double roots ⇒ the system is just oscillating. (Critical damped regime).
- $0<\zeta<1 \Rightarrow p_1$ and p_2 are two complex roots ⇒ the system is unstable (it oscillates: damping on critical).

IV.4.2 Response of second order systems (Responses to a step input)

Consider a second-order linear system. The input to the system is a step $e(t) = 1$.

$$S(p) = \frac{1}{p(T^2p^2 + 2\zeta Tp + 1)}$$

IV.4.2.1 step response for ζ>1

We are talking about a high damping system. The two real poles p_1 and p_2 give an answer which will be the sum of two exponentials.

$$s(t) = 1 + \frac{1}{2}\frac{\zeta}{\sqrt{\zeta^2 + 1}}e^{-(\zeta - \sqrt{\zeta^2 + 1})\omega_n t}$$

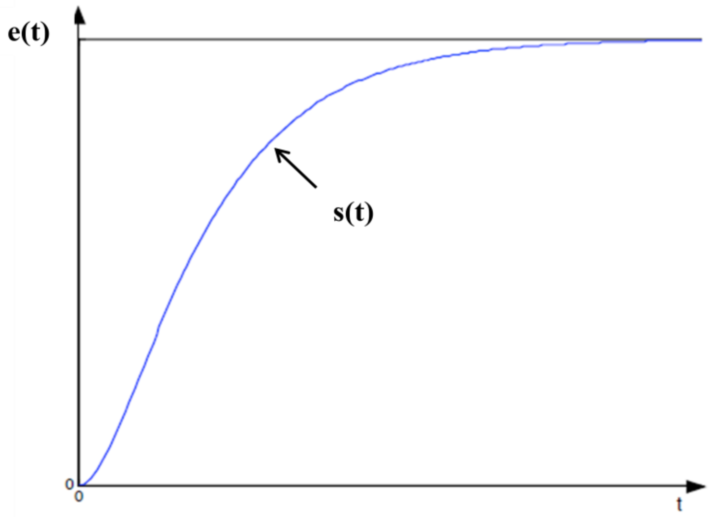


Figure IV.8: Second-order index response with strong damping

The characteristics of this response are:

- The steady state is $s_P(t) = KE_0$.
- At the origin, the tangent is horizontal.

IV.4.2.2 step response for ζ=1

The system is in critical damping, $s(t) = 1 - e^{-\omega_n t}(1 + \omega_n t)$.

The response curve resembles the curve obtained in the previous paragraph, but the growth is faster.

IV.4.2.3 Response at scale for ζ < 1

We are talking about a low damping system.

$$s(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\sqrt{1-\zeta^2}\,\omega_n t + \varphi)$$

With

$$\varphi = arctg(\frac{\zeta}{\sqrt{1-\zeta^2}})$$

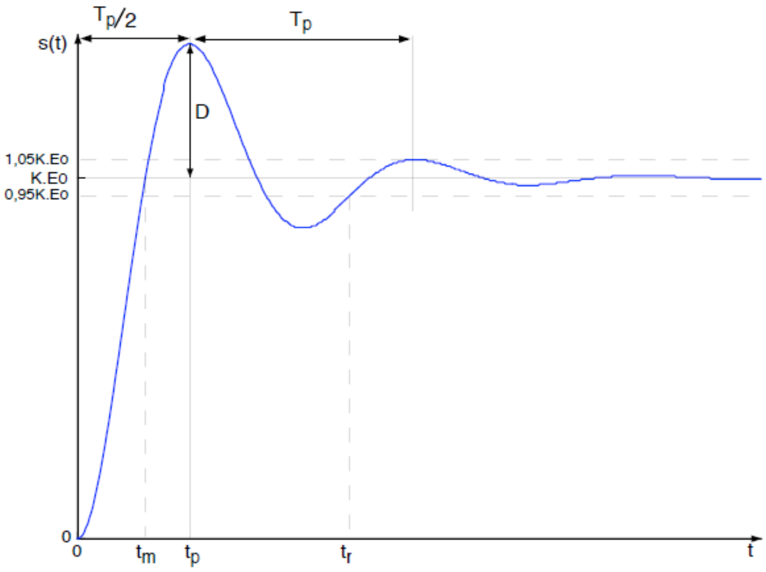


Figure IV.9: Second-order index response with low damping

The characteristics of this response are:

- Steady state $sp(t) = KE_0$
- At the origin, the tangent is horizontal
- Propre pulsation amortie: $\omega_p = \omega_n\sqrt{1-\zeta^2}$
- Pseudo-period of oscillations : $T_p = \frac{2\pi}{\omega_p}$
- T_m is the time it takes for the response to a step to be at 90% of the final value, and also $T_m$and the rise time (time after which s(t) first reaches sp (t)) $T_m = \frac{T_p}{2}(1 - \frac{\varphi}{\pi})$.
- t_p : Peak time $t_p = \frac{T_p}{2} = \frac{\pi}{\omega_p}$
- t_r: response time at 5% is the time after which the output reaches steady state to within 5% and remains there. The attached chart gives this time according to the characteristics of the transfer function. an approximation for $\zeta \ll 1$ is

  $$t_p = 3\frac{\tau_n}{\zeta} = \frac{3}{\zeta\omega_n}$$

- Overtaking $D = s(t_p) - KE_0$ the calculation gives $D = KE_0 e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$

  We can also define the relative overrun (without units):

  $$D_r = \frac{D}{KE_0} = e^{\frac{-2\zeta\pi}{\sqrt{1-\zeta^2}}}$$

- Successive overruns: the ratio between two successive overruns of the same sign can make it possible to identify the damping ζ

$$ln\frac{D_2}{D_1} = \frac{-2\zeta\pi}{\sqrt{1-\zeta^2}}$$

IV.4.2.4 Resonance:

When the transfer function T_{dB} is maximum, the resonance frequency is equal to:

$\omega_R = \omega_n\sqrt{1-2\zeta^2}$; ω_R exists if $1\text{-}2\zeta^2 > 0 \Rightarrow \zeta \leq 0.7$

The overvoltage coefficient is equal to: $Q = \frac{1}{2\eta\sqrt{1-\zeta^2}}$

- $\zeta < 0.7$ => Oscillating systems (Oscillations are visible).
- $\zeta < 0.7$ => No oscillations

IV.5 Exercises

Exercise 1

Consider a system written by the following function diagram:

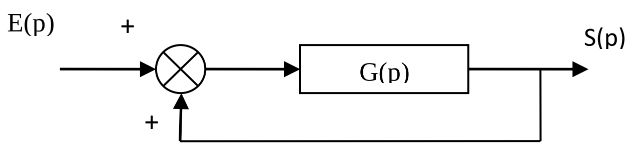


Figure IV.10: Representation of a closed loop with unitary feedback

With:

$$G(p) = \frac{k}{p(p+2)}$$

- Find the ranges of variations of k for the three possible regimes.

Solution :

$$T(p) = \frac{G(p)}{1+G(p)}$$

$$T(p) = \frac{\frac{k}{p(p+2)}}{1+\frac{k}{p(p+2)}} = \frac{k}{p^2+2p+k} = \frac{\omega_n^2}{p^2 2\eta\omega_n + \omega_n^2}$$

$$\Rightarrow \begin{cases} \omega_n^2 = k \\ 2\eta\omega_n = 2 \end{cases} \Rightarrow \eta = \frac{1}{\omega_n} = \frac{1}{\sqrt{k}}$$

For 0 <η< 1 $\Rightarrow \frac{1}{\sqrt{k}} < 1 \Rightarrow$ k > 1 Damping regime on critical.

For η = 1 $\Rightarrow \frac{1}{\sqrt{k}} = 1 \Rightarrow$ k = 1 Critical cushioned regime.

For η> 1 $\Rightarrow \frac{1}{\sqrt{k}} > 1 \Rightarrow$ k < 1 Regime on cushioning

Exercise 2

Let the following transfer function be:

$$T(p) = \frac{2K}{(p+2k)(p+1)}$$

1- For what value of K is the system in critical damping mode.
2- For $\zeta = 0.5$.

Calculate: The overvoltage coefficient Q, ω_R , and overtaking D

Solution :

1- Critical damping: $\Rightarrow \zeta = 1$

$\omega_n = 1, K = 0.5$

2- For $\zeta = 0.5$

The overvoltage coefficient is equal to: $Q = \frac{1}{2\eta\sqrt{1-\eta^2}}$

$Q = 1.15$

ω_R: Does not exist because k complex.

$$D = \exp\left(\frac{-\pi\eta}{\sqrt{1-\zeta^2}}\right)$$

$D = 0.16$

IV.6 Additional exercises

Exercise 1

Consider a system having the following transfer function:

$$T(p) = \frac{K}{(1 + T_p)}$$

We inject the system, in open loop, with an amplitude step U = 5 and its response is obtained in figure. IV.11

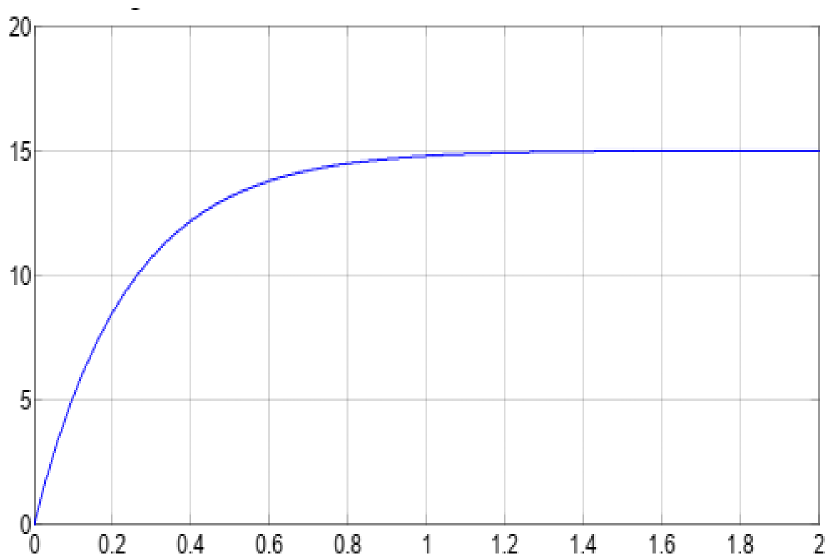


Figure IV.11: Index response of a 1st order system without delay in BO

1- Find the values of K and T

Exercise 2

Consider a system having the following transfer function:

$$T(p) = \frac{K}{(1 + T_p)}$$

We inject into the system, in open loop, an amplitude step U = 2 and its answer is obtained in the figure. IV.12

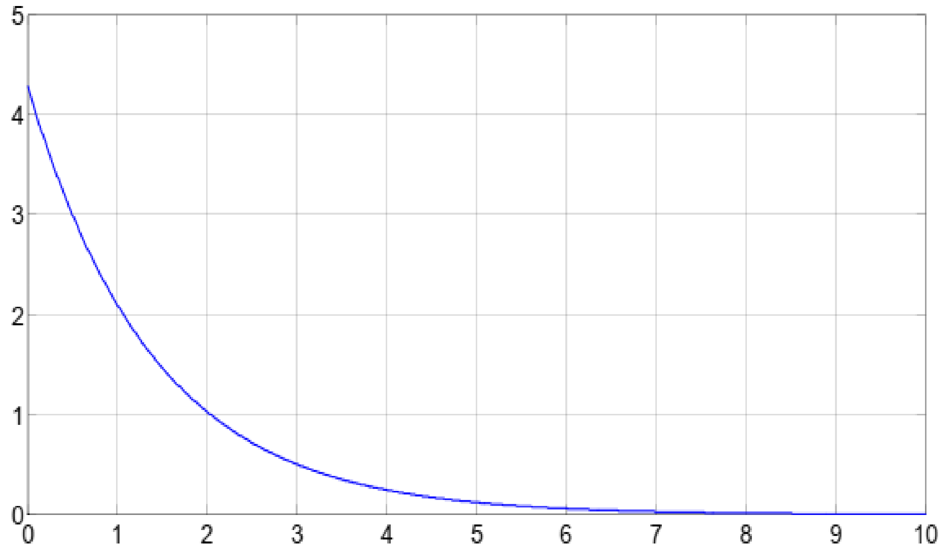


Figure IV.12: Step response of a 1st order system with decreasing response

1- Find the values of K and T

Exercise 3

We study the dynamic behavior of a mechanical mass-spring-damper system. Its unitary step response is shown in figure IV.13.

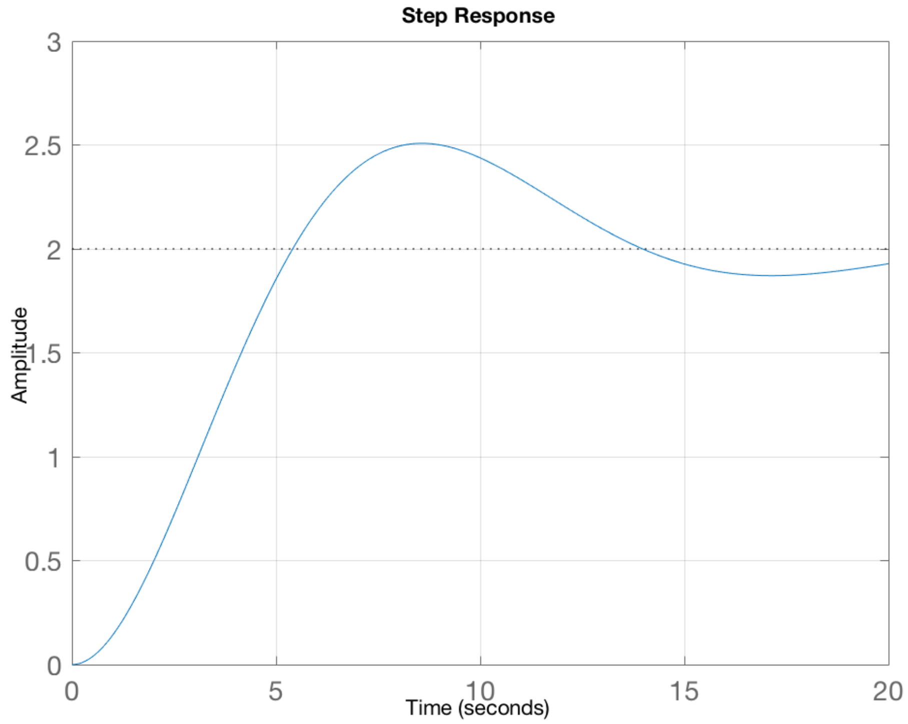


Figure IV.13: Unitary index response of the mechanical system

1- Propose by justifying a model in the form of a transfer function $G(p) = \frac{Y(p)}{U(p)}$

2- Determine its static gain, its first overshoot moment and its first overshoot value.

3- Deduce the damping coefficient and the undamped natural pulsation

Chapter V :

Stability of servo systems

V. Stability of servo systems

V.1 Introduction

The study of stability is of paramount importance in the study of systems and servo systems. In this chapter we will give the different stability criteria of a control.

V.2 Definition

A stable system can be defined as a system that remains at rest unless excited by an external source and that returns to rest as soon as all excitation ceases. A stable system can be defined as a system whose impulse response tends to zero when t tends to infinity. In automatic mode, we will define stability by one of the following propositions: A linear system is stable:

- When its response to a step takes a finite value in steady state.
- When its response to an impulse tends towards zero.
- When its response has a sinusoid is a sinusoid of finite amplitude.

V.3 Condition of stability of a looped system

A linear system is stable under the necessary condition that all poles of the transfer function have a negative real part.

- If the real part of one of the poles of the closed-loop transfer function is zero, $\Rightarrow$ the system is just oscillating.
- If the real parts of one of the poles of the closed-loop transfer function is less than zero $\Rightarrow$ stable system.
- If the real part of one of the poles of the closed-loop transfer function is greater than zero $\Rightarrow$ unstable system.

V.4 Stability criteria

There are three stability analysis criteria.

V.4.1 Mathematical criterion:

Consider the general diagram of a controlled system shown in figure V.1, the mathematical condition of stability is stated as follows:
A servo system is stable if and only if its closed-loop transfer function (FTBF) does not have any positive real part poles.

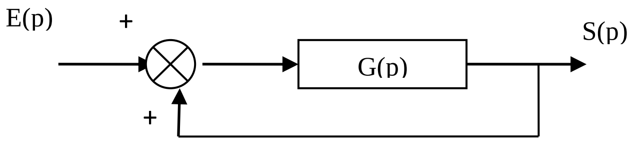


Figure V.1: Closed loop system with unitary feedback

Example:

Let the closed-loop transfer function system be given by:

$$F(p) = \frac{k}{(p+5)(p+2)}$$

The poles of the system are:

$p_1 = -5 < 0, p_2 = -2 < 0$ ⇒ Two poles with negative real parts ⇒ stable system.

V.4.2 Algebraic criteria:

This criterion is applicable to the characteristic equation of a closed-loop system.

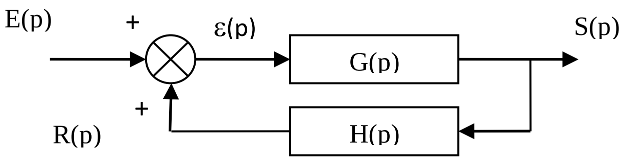


Figure V.2: Closed loop system

$$F(p) = \frac{G}{1+GH}$$

1+GH = 0 is the characteristic equation.

V.4.2.1 ROUTH criterion:

It is a criterion which allows us to know if the roots of the characteristic equation have their negative parts without having to solve them.

a) Necessary and sufficient conditions:

The ROUTH criterion is only applicable if all the Ai of the algebraic equation are positive.

b) Construction of the ROUTH table:

Let the characteristic equation be: $1+T(p) = A_m P^m + A_{m-1} P^{m-1} + \dots + A_1 P + A_0 P^0 = 0$.

We arrange the coefficients on the line.

	$A_m P^m + A_{m-1} P^{m-1} + \dots + A_1 P + A_0 P^0 = 0$		
P^m	A_m	A_{m-2}	A_{m-4}
P^{-1m}	A_{m-1}	A_{m-3}	A_{m-5}
P^{m-2}	b_1	b_2	b_3
P^{m-3}	c_1	c_2	c_3
P^0	d_1	d_2	d_3

$$b_1 = \frac{A_{m-1}A_{m-2} - A_m A_{m-3}}{A_{m-1}}\ ; \quad b_2 = \frac{A_{m-1}A_{m-4} - A_m A_{m-5}}{A_{m-1}}$$

$$c_1 = \frac{b_1 A_{m-3} - A_{m-1} b_2}{b_1}\ ; \qquad c_2 = \frac{b_1 A_{m-5} - A_{m-1} b_3}{b_1}$$

A system is stable if all elements in the first column of the ROUTH table are positive. If a coefficient in this line is zero or negative, then the system is unstable.

Getting zeros on an entire line corresponds to a pair of imaginary conjugate poles.

Example 1:

$E(p) = p^4 + 6p^3 + 13p^2 + 12p + 4 = 0$

The corresponding Routh table is as follows:

p^4	1	13	4
p^3	6	12	0
p^2	11	4	0
p^1	9.82	0	0
p^0	4	0	0

There is no change of sign in the first column, so the FTBF does not have poles with a positive real part; consequently, the controlled system is stable in a closed loop.

Example 2:

Either the following characteristic equation:

$E(p) = p^5 + 6p^4 + 12p^3 + 12p^2 + 11p + 6 = 0$

p^5	1	12	11
p^4	6	12	6
p^3	10	10	0
p^2	6	6	0
p^1	0	0	0
p^0	12	0	0

A line with all zeros implies 1 pair of pure imaginary poles.

$E_1(p) = 6p^2 + 6$

And we replace in the following line with:

$\frac{dE_1(p)}{dp} = 12p$

And constitutes the Routh table.

There is no change of sign in the coefficients of the first column, so the FTBF does not have poles with a positive real part, but it has two pure imaginary poles. These are the roots of:

$E_1(p) = 6p^2 + 6 = 0 => p = \pm j$

Noticed:

The Routh criterion is very useful when the coefficients of the polynomial are servo adjustment parameters.

Example 3:

Either a system placed in a control loop with unitary feedback figure V.1,

$$(p) = \frac{K}{p(p^2 + p + 3)}$$

Using Routh's criterion, find the value of K for which the closed-loop system is stable.

The system is stable for: K<3.

Noticed:

The mathematical criterion and that of Routh, are criteria of absolute stability, they do not make it possible to specify the margins of stability of the system. In other words, they do not indicate the degree of stability or instability.

V.4.2.2 Hurwitz criterion (The Hurwitz determinant)

Let the characteristic equation:

$$A_m P^m + A_{m-1} P^{m+1} + \cdots + A_1 P^1 + A_0 P^0 = 0$$

The Hurwitz criterion is only applicable if all the Ai are positive.

$$\begin{vmatrix} A_{m-1} & A_{m-3} & A_{m-5} & \dots\dots \\ A_m & A_{m-2} & A_{m-4} & \dots\dots \\ 0 & A_{m-1} & A_{m-3} & \dots\dots \\ 0 & A_m & A_{m-2} & \dots\dots \\ . & & & \end{vmatrix}$$

Δ_1 Δ_2 Δ_3

A controlled system is stable only if all the bi are positive (necessary condition) and all the determinants obtained are strictly positive ($\Delta i > 0$), this means that all the poles of the system have their negative real part.

-If one of the determinants Δi is zero, implies that the system is at the limit of stability.

-If one of the determinants Δi is zero, this implies that the system is unstable.

Example: Consider a linear servo system whose characteristic equation is given by:

$$p^3 + 2p^2 + 4p + 6 = 0$$

Study the stability of the system according to the Hurwitz criteria.

Solution:

-We notice that all the coefficients bi are positive => the necessary condition is verified.

-We calculate the Hurwitz determinants:

$$\Delta_3 = \begin{vmatrix} 2 & 6 & 0 \\ 1 & 4 & 0 \\ 0 & 2 & 6 \end{vmatrix} = 12 > 0;\ \Delta_2 = \begin{vmatrix} 2 & 6 \\ 1 & 4 \end{vmatrix} = 2 > 0\ et\ \Delta_1 = 2 > 0$$

As all Δi are positive, then the system is stable.

V.5 Exercises

Exercise 1

Let the system be described by:

$$T(p) = \frac{K}{(1+p)(2+p)(5+p)}$$

1- For what value of K the system is stable.

2- Check stability using ROUTH criteria.

Solution:

Characteristic equation: $p^3 + 8p^2 + 17p + 10 + p = 0$

$$\begin{array}{c|cc}
p^3 & 1 & 17 \\
p^2 & 8 & 10+k \\
p^1 & 126-k & 0 \\
p^0 & 10+k &
\end{array}$$

The system is stable if: $\begin{cases} 126 - \mathrm{k} > 0 \\ 10 + \mathrm{k} > 0 \end{cases}$

$\Rightarrow \; 10 < k < 126$

Exercise 2

Let the system be written by:

$$T(p) = \frac{K}{(3+p)(1+Ap)}$$

1- For what value of K the system is stable.

2- Check stability using ROUTH criteria.

Solution:

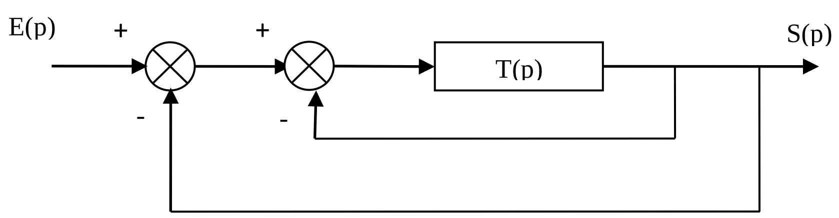
Characteristic equation: $Ap^2 + (3A + 1)p + 3 + k = 0$

$$
\begin{array}{c|ll}
p^2 & A & 3+k \\
p^1 & 3A+1 & 0 \\
p^0 & 3+k &
\end{array}
$$

The system is stable if: $\begin{cases} A > 0 \\ 3 + k > 0 \end{cases} \Rightarrow \begin{cases} A > 0 \\ k > -3 \end{cases}$

Exercise 3

Consider a slave system described by the following diagram:

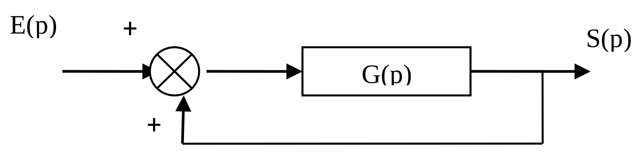


$$T(p) = \frac{K}{(1 + 2p)\,(1 + p)^3}$$

1- Check stability using ROUTH criteria

Solution:

$$G(p) = \frac{T(p)}{1 + T(p)}$$



$$F(p) = \frac{H(p)}{1 + H(p)}$$

The characteristic equation of the transfer function $F(p)$:

$$2p^4 + 7p^3 + 9p^2 + 5p + 1 + 2k = 0$$

p^4	2	9	1+2k
p^3	7	5	0
p^2	7.57	1+2k	0
p^1	4.07-2k	0	0
p^0	1+2k	0	

The system is stable if: $4.07 - 2k > 0$ and $1 + 2k > 0 \Rightarrow -0.5 < k < 2.03$

Exercise 4

Consider the two functions of the transfer of a slave system:

$$F(p) = \frac{1}{p^4 + 7p^3 + 17p^2 + 17p + 6}$$

$$H(p) = \frac{1}{p^4 + 2p^3 + 3p^2 + 4p + 5}$$

1- Check the stability of this system.

Solution:

$$F(p) = \frac{1}{p^4 + 7p^3 + 17p^2 + 17p + 6}$$

p^4	1	17	6
p^3	7	17	0
p^2	14.57	6	0
p^1	14.12	6	
p^0	6	0	

Note that all the coefficients in the first column have the same sign (positive).

The system is stable.

$$H(p) = \frac{1}{p^4 + 2p^3 + 3p^2 + 4p + 5}$$

p^4	1	3	5
p^3	2	4	0
p^2	1	5	0
p^1	-6	0	
p^0	5	0	

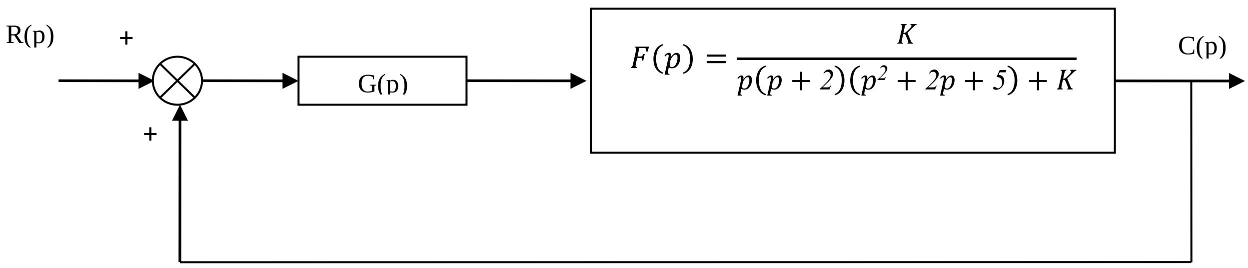
We note that all the coefficients in the first column do not have the same sign. Indeed, there are two changes of sign (from +1 to -6 and from -6 to +5), this means that the system has two poles with a positive real part (two unstable poles), therefore the system is unstable.

Exercise 5

In the figure below, the closed loop system.

1- Determine the closed loop transfer function $F(p) = \frac{C(p)}{R(p)}$

2- Determine the range of values of K which allows the looped system to be stable.



Solution:

1- The transfer function $F(p) = \frac{C(p)}{R(p)}$:

$$F(p) = \frac{K}{p(p+2)(p^2+2p+5)+K}$$

2- The range of values of K which allows the looped system to be stable

$D(p) = p^4 + 4p^3 + 9p^2 + 10p + K = 0$

p^4	1	9	k
p^3	4	10	0
p^2	6.5	k	0
p^1	(65-4k)/6.5	0	0
p^0	k	0	

The system is stable if: $65 - 4k > 0$ et $k > 0 \Rightarrow 0 < k < 16.25$

V.6 Additional exercises

Exercise 1

Using the ROUTH criterion, discuss the position of the roots of the following polynomials in the complex plane:

$a)$ $p^4 + 10p^3 + 35p^2 + 50p + 24 = 0$

$b)$ $p^3 - 4p^2 - 7p + 10 = 0$

$c)$ $p^5 + 2p^4 + 15p^3 + 35p^2 - 17p - 20 = 0$

Exercise 2

We consider the following polynomial:

$4p^4 + 9p^3 + 3p^2 + 2p + k = 0$

What is the condition on k so that all the roots of this polynomial are on the left of the complex plane C ?

Exercise 3

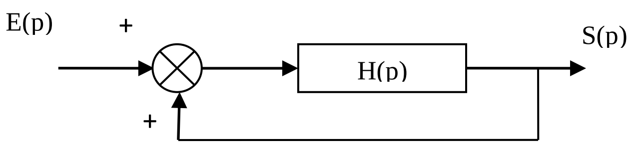
Let F(p) be the transfer function of an open-loop controlled system:

$$F(p) = \frac{25}{(p+1)(p+3)(p+7)}$$

- Check stability using ROUTH criteria

Exercise 4

Consider the following system:



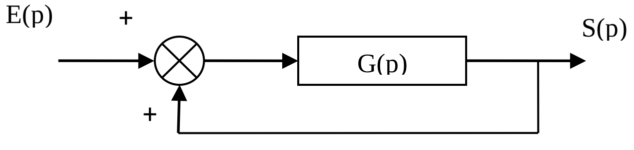
$$H(p) = \frac{3K}{p.(p+1)(p+3)}$$

- Find the condition on k for the system to be stable.

Exercise 5

Consider a system placed in a unitary feedback regulation loop with:

$$G(p) = \frac{3K}{p.(p+1)(p+3)}$$



- Find the value of K so that the system is stable.

Exercise 6

F(p) the transfer function of a closed loop controlled system:

$$G(p) = \frac{K(p+10)}{p^3 + 4p^2 + (4+K)p + 10K}$$

Find the value of K so that the system is stable.

Exercise7

Consider a cascaded proportional regulator with a system:

$$G(p) = \frac{10K}{(4p+1)(2p+1)^2}$$

1- Determine the values of K to have a closed loop table system is stable (according to Routh-Hurwitz)

Chapter VI :

Frequency analysis of linear systems

VI. Frequency analysis of linear systems

VI. 1 Frequency response

The objective of frequency analysis is to study the behavior and response of a linear system to a sinusoidal load. The output of the linear system will be sinusoidal with the same pulsation as the input signal but of different amplitude and out of phase with the input signal. The interest of frequency study is to then be able to know the behavior of a system in the face of any signal by breaking down the signal into Fourier series.

Rich in information, it provides a link between the transfer function of the system considered, and its response to a sinusoid. This response will be characterized by two parameters, the gain and the phase shift. The sinusoidal transfer function is obtained by identifying p=jω in the transfer function G(p); it is made up of all the sinusoidal transfer functions when the pulsation varies from 0 to infinity.

When the input signal is sinusoidal, we will not use the Laplace formalism, we will use the complex transfer function where the operator p is replaced by (jω) (ω pulsations of the input signal)

$$H(j\omega) = \frac{S(j\omega)}{E(j\omega)}$$

VI. 2 BODE diagram

The Bode diagram consists of plotting the open loop transfer function, a gain curve for the module and a phase curve for the argument.
we consider the first order system :

$T(p) = \frac{1}{1+\tau p}$, We pose p = j ω $\Rightarrow T(p) = \frac{1-j\tau\omega}{1+\tau^2\omega^2} = \frac{1}{1+\tau^2\omega^2} - j\frac{\tau\omega}{1+\tau^2\omega^2}$;

$\frac{1}{1+\tau^2\omega^2}$ Part Real, $\frac{-\tau\omega}{1+\tau^2\omega^2}$ Imaginary Part.

The modulus of T(p): $|T(p)| = \sqrt{\mathrm{Re}^2 + \mathrm{Im}^2} = \frac{1}{\sqrt{1+\tau^2\omega^2}} = \left(1+\tau^2\omega^2\right)^{-\frac{1}{2}}$

The phase of $T(p)$: $tg\varphi = \frac{\mathrm{Im}}{\mathrm{Re}} = -\tau\omega \Rightarrow \varphi = -artg\,\tau\omega$

φ : Phase = Argument.

From the transfer loci which represent the variations of the module and the phase of the transfer function of a system as a function of the frequency (ω), we can predict the stability of closed loop systems from their functions of open loop transfer.

From $\begin{cases} |T(j\omega)| \\ \varphi(j\omega) \end{cases}$ in Open Loop $\Rightarrow$ Stability of the Closed-Loop system.

The frequency response (Hz) consists of plotting separately against the pulsation ω (rd/s).
The module (20log $|T(j\omega)|$) and the phase of this response.
The amplitude curve or gain curve is obtained from the module.
The phase curve is obtained from the argument.
These curves make it possible to carry out stability measurements and to define the gain and phase margins which represent the safety margins of a controlled system. We use the semi-logarithmic scale to trade gain and phase curves.

VI. 3 Definition of the semi-logarithmic scale

A semi-logarithmic mark is a mark in which one of the axes is graduated according to a linear scale, while the other axis is graduated according to a logarithmic scale.

The distance between 1 and 10 is the same as the distance between 10 and 100 and the distance between 0.1 and 1 because log (100) – log (10) = log (10) – log (1) = log (1) – log (0,1).

Each of these intervals is called a module or decade. the distance which separates 1 from 2 is equal to that which separates 10 from 20 but is greater than that which separates 2 from 3 because log(2) - log(1) = log(20) - log(10) > log(3) - log(2). The unit for the gain plot and the Decibel “dB”. For the phase we use a normal ordinate.

VI. 4 Examples

Example 1:

$$T(p) = (1 + j\tau\omega)^n$$

$$T(j\omega) = (1 + j\tau\omega)^n \Rightarrow \begin{cases} |T(j\omega)| = \left(1^2 + (\tau\omega)^2\right)^{\frac{n}{2}} \\ \varphi = n \quad arctg\dfrac{\tau\omega}{1} = n \quad arctg\,\tau\omega \end{cases}$$

Cutoff frequency:

T_{dB}= 20 log $|T_c|$ =0 $\Rightarrow T_{dB}$= 20 log $\left(1 + (\tau\omega)^2\right)^{\frac{1}{2}} \approx 20\log(\tau\omega)$

pour ω>>0 $\Rightarrow \omega_c = \dfrac{1}{\tau}$

Calculation of T_{dB} limits

•$\omega = \omega_c \Rightarrow T_{dB}$= 20 log $|T_c|$ = 0

•$\omega \to 0^+ \Rightarrow T_{dB}$= 20 log $\left(1 + (\tau\omega)^2\right)^{\frac{1}{2}} \to 0^+ \Rightarrow (\tau\omega) \to 0^+ \Rightarrow T_{dB}$= 0

Et $\Rightarrow \varphi = n\ arctg\,\tau 0 \Rightarrow \varphi = 0$

•$\omega \to +\infty \Rightarrow T_{dB}$= 20 log $\left(1 + (\tau\omega)^2\right)^{\frac{1}{2}} \approx 20\log(\tau\omega) T_{dB} \to n\,\infty$

Et $\Rightarrow \varphi = n\ arctg\,\tau\infty \Rightarrow \varphi = n\dfrac{\pi}{2}$

ω (rd/s)	$\omega<\omega_c$	$\omega>\omega_c$
T_{dB}	0	$\to n\,\infty$
φ	0	n π/2

Calculation of the slope:

It takes two points to trace the slope.

First point: $\omega = \omega_c \Rightarrow T_{dB}$= 20 log $\left(1 + (\tau\omega)^2\right)^{\frac{1}{2}} \approx 20\log(\tau\omega_c)$= 0

Second point: $\omega' = 10\,\omega_c \Rightarrow T_{dB}$= n 20 log$(\tau\omega')$ =n 20 log $(10\tau\omega_c)$ =n 20_{dB}

Slope: n 20_{dB} by decade.

Example 2:

Consider a system with a transfer function $T(p) = \frac{1}{\tau p}$, and $\alpha = -1$, we pose $p = j\omega$

$T(j\omega) = (j\tau\omega)^{-1} \Rightarrow \{|T(j\omega)| = (\tau\omega)^{-1}$

$$\varphi = arctg\frac{I_m}{R_e} = arctg\frac{(\tau\omega)^{-1}}{0} = -\frac{\pi}{2}$$

Calculation of the cutoff frequency ω_c at $T_{dB} = 0$:

$$TdB = 20\log|T(j\omega)| = 0 \Rightarrow 20logT(\tau\omega_c) = 0 \Rightarrow \tau\omega_c = 1 \Rightarrow \omega_c = \frac{1}{\tau}$$

$\omega \to 0^+ \Rightarrow TdB = 20log|T(j\tau\omega)| = 20log(\tau\omega)^{-1} = -20log(\tau\omega) \to +\infty;$

$\omega \to +\infty \Rightarrow TdB = 20log|T(j\tau\omega)| = -20log(\tau\omega) \to -\infty;$

$\omega \to \omega_c \Rightarrow TdB = 20log|T(-j\tau\omega)| \to 0$

Example 3: $T(p) = (1 + \tau p)^{\beta}$

We will study this function with $\beta = 1\ et\ \beta = -1$

-Consider a system with a transfer function

$T(p) = (1 + \tau p) \Rightarrow T(j\omega) = (1 + \tau j\omega)\ et\ \beta = 1$ on pose $p = j\omega$

$T(j\omega) = (1 + \tau j\omega) \Rightarrow \{|T(j\tau\omega)| = (1^2 + (\tau\omega)^2)^{\frac{1}{2}}$ et $\varphi = arctg\frac{\tau\omega}{1} = arctg\tau\omega$

$$TdB = 20log|T(j\tau\omega_c)| = 0 \Rightarrow \omega_c = \frac{1}{\tau}$$

The gain curve:

$AdB = 20log|T(j\omega)| = 20log\sqrt{1 + (\tau\omega)^2}$

$\omega \to 0 \Rightarrow AdB = 0$: Premier Asymptote

$\omega \to +\infty \Rightarrow AdB = 20\log\tau\omega \Rightarrow 20\ dB/déc$: Deuxième Asymptote

$\omega \to \omega_c \Rightarrow AdB = 20log\sqrt{2} = 10log2 \Rightarrow AdB = 3dB$

The phase curve:

$$\varphi = arctg\frac{I_m}{R_e} = arctg\frac{\tau\omega}{1}$$

$\omega \to 0 \Rightarrow \varphi = arctg\frac{0}{1} \Rightarrow \varphi = 0$: Premier Asymptote

$\omega \to +\infty \Rightarrow \ \varphi = arctg\frac{+\infty}{1} \Rightarrow \varphi = \frac{\pi}{2}$: Deuxième Asymptote

$$\omega \to \omega_c \Rightarrow \ \varphi = arctg\frac{1}{1} \Rightarrow \varphi = \frac{\pi}{4}$$

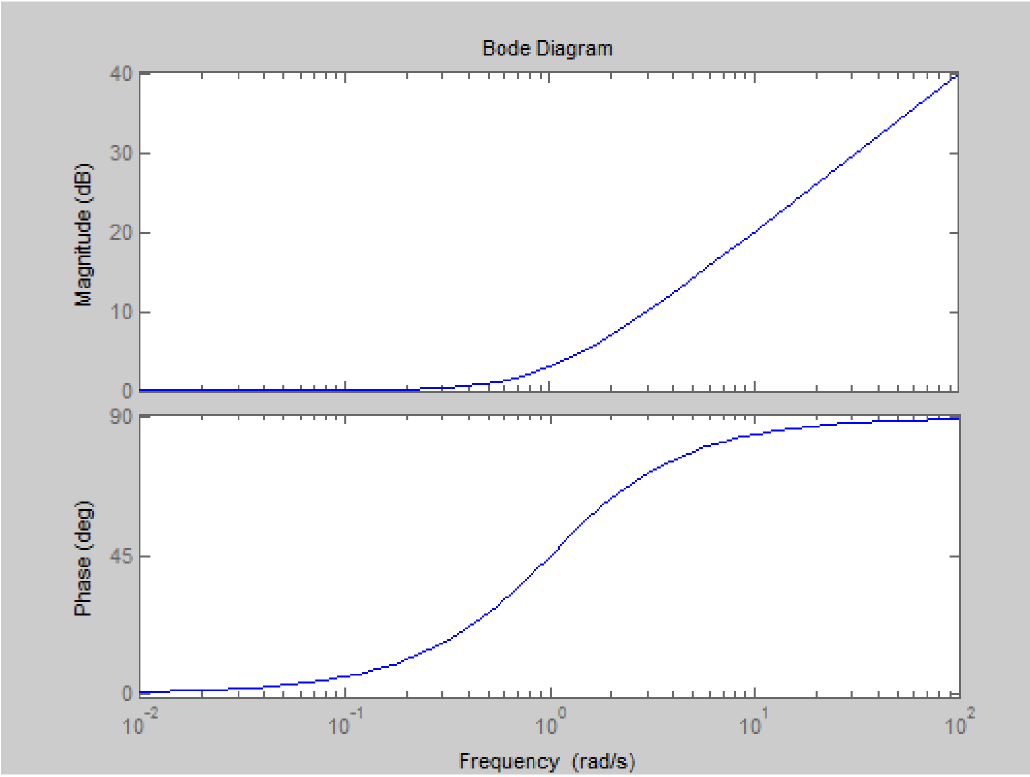


Figure VI.1: Bode curve of a 1st Order type system for β=1

We consider system with a transfer function

$$T(p) = \ (1 + \tau p)^{-1} \Rightarrow T(j\omega) = (1 + \tau j\omega)\ and\ \beta = 1, p = j\omega$$

The gain curve:

$$AdB = -20log|T(j\omega)| = -20log\sqrt{1 + (\tau\omega)^2}$$

$\omega \to 0 \Rightarrow AdB = 0$: Premier Asymptote

$\omega \to +\infty \Rightarrow \ AdB = -20\log\tau\omega \Rightarrow -20\ dB/déc$: Deuxième Asymptote

$\omega \rightarrow \omega_c \Rightarrow \; AdB = -20log\sqrt{2} = -10log2 \Rightarrow AdB = -3dB$

The phase curve:

$$\varphi = -arctg\frac{I_m}{R_e} = -arctg\frac{\tau\omega}{1}$$

$\omega \rightarrow 0 \Rightarrow \varphi = -arctg\frac{0}{1} \Rightarrow \varphi = 0$: Premier Asymptote

$\omega \rightarrow +\infty \Rightarrow \; \varphi = -arctg\frac{+\infty}{1} \Rightarrow \varphi = -\frac{\pi}{2}$: Deuxième Asymptote

$$\omega \rightarrow \omega_c \Rightarrow \; \varphi = -arctg\frac{1}{1} \Rightarrow \varphi = -\frac{\pi}{4}$$

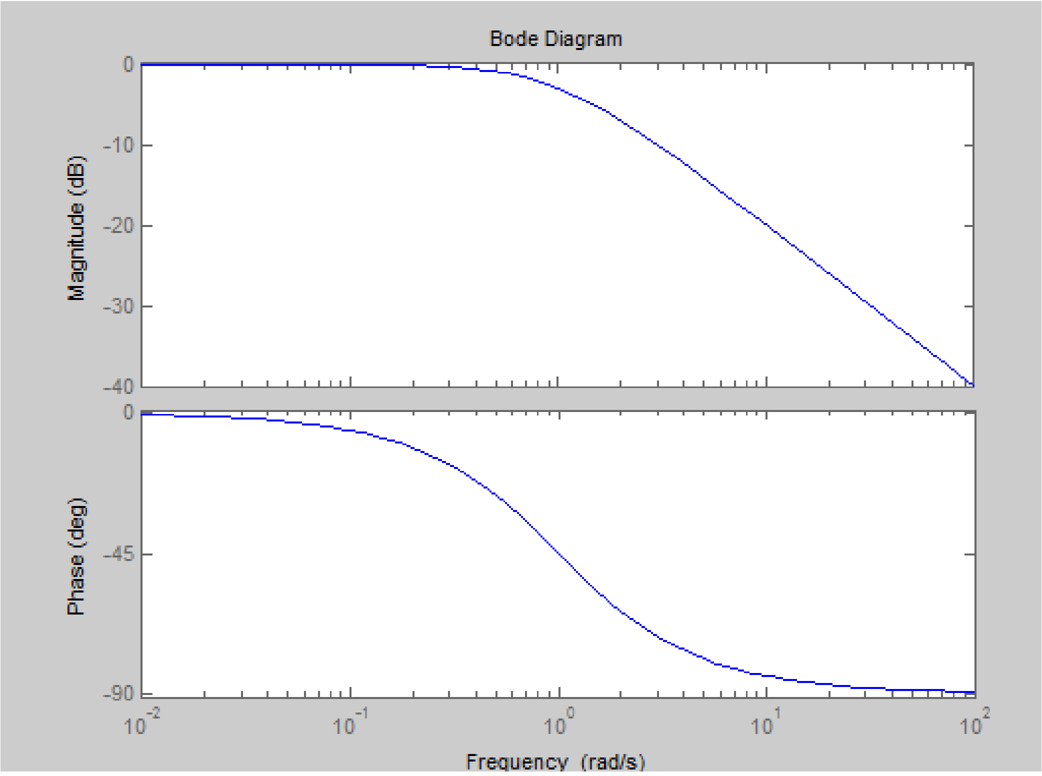


Figure VI.2: Bode curve of a 1st Order system of type for β=-1

VI. 5 Nyquist diagram

It is a graphical criterion of stability in Closed Loop obtained from the Nyquist locus of the system in Open Loop. It uses Cauchy's theorem applied to the transfer function of the servo system.

VI.5.1 Nyquist contour

It is defined by the half-perimeter of a circle of radius R and center 0 when $R \to \infty$, on the side of the positive real parts.

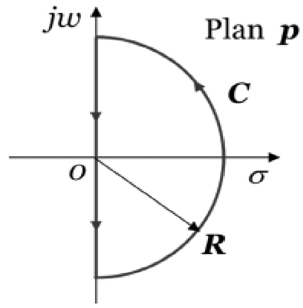


Figure VI.3: Nyquist contour

A system is stable in a closed loop if the image of the Nyquist contour by $E(p) = 1 + G(p)$ makes around the origin, in the trigonometric direction, a number of turns equal to the number of poles at part positive real of the open loop transfer function $G(p)$.

A system is stable in a closed loop if made around the critical point, in the trigonometric direction, a number of turns equal to the number of poles with positive real part of the open loop transfer function $G(p)$.

Note: the study of the image of C by $E(p) = 1 + G(p)$ with respect to the origin (0,0) is equivalent to the study of its image by $G(p)$ with respect to at the point (-1.0) called the critical point.

Hence the statement following the image of the Nyquist contour by the function G(p).

VI.5.2 Reverse Criteria:

If the FTBO of a controlled system does not have any pole with a positive real part, then this system is stable in a closed loop if, by traversing the Nyquist locus of the FTBO in the direction of increasing ω (from 0 to ∞), we leaves the critical point (-1.0) to the left of the curve.

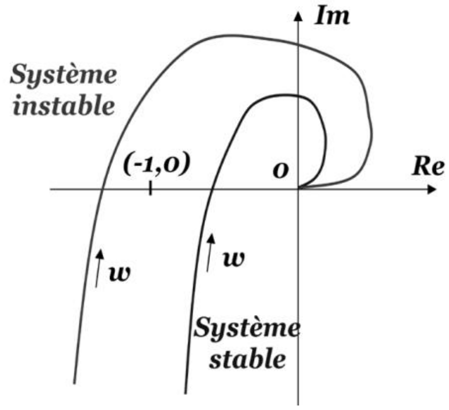


Figure VI.4: Reversal criterion from the Nyquist locus

We must not forget that we always trace the Nyquist locus of the system in open loop to study the stability in closed loop.

Nyquist analysis consists of a graphical process for determining variations in the modulus and phase of the transfer function.

Constitution of Nyquist diagrams: For a transfer function T(p) relating to an open loop system, the Nyquist diagram is the locus of the defined points:

a- In polar coordinats:

Defined by a radius vector equal to the value of the modulus of $T(j\omega)$ and a polar angle equal to the argument of $T(j\omega)$.

The location is scaled with respect to the frequency ω.

$$T(j\omega) = |T(j\tau\omega)|e^{j\varphi} = |T(j\omega)|(cos\varphi + jsin\varphi)$$

b- In rectangular coordinats:

Defined by a curve giving the variation of the imaginary part as a function of the real part of the transfer function.

$$I_m\big((j\omega)\big) = f(R_e(T(j\omega))$$

Example:

Consider a system with a transfer function

$$T(p) = \ (1+p)^{-1}, on\ pose\ p = j\omega \Rightarrow T(j\omega) = (1+j\omega)^{-1}$$

-In polar coordinates:

$$T(j\omega) = (1+j\omega)^{-1} \Rightarrow \{|T(j\tau\omega)| = -\sqrt{1+(\omega)^2}\ et\ \ \varphi = -arctg\frac{\omega}{1} = -arctg\omega$$

-In rectangular coordinates

$$T(j\omega) = \frac{1}{1+j\omega} = \frac{1}{1+\omega^2} - j\frac{\omega}{1+\omega^2} = R_eT(j\omega) + I_mT(j\omega)$$

$$\omega \to 0 \Rightarrow R_e = 1\ et\ \ I_m = 0;$$

$$\omega \to \infty \Rightarrow\ R_e = 0\ et\ I_m = 0;$$

$$\omega \to 1 \Rightarrow R_e = 0{,}5\ et\ I_m = -0{,}5$$

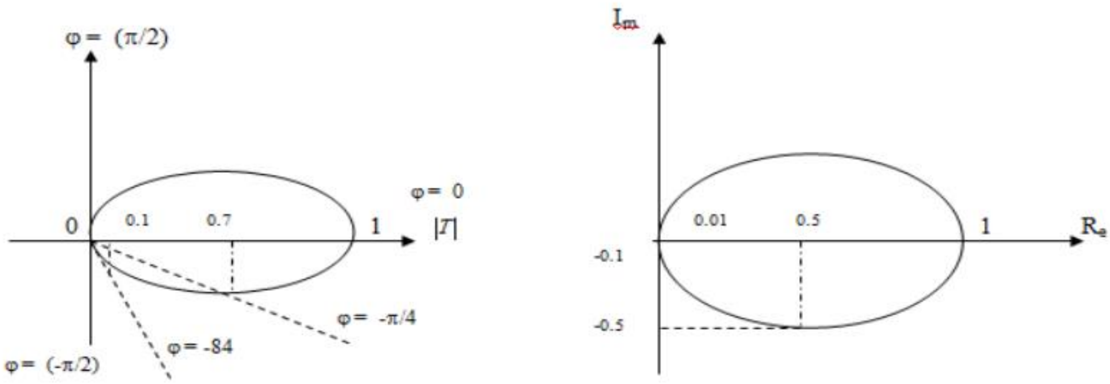


Figure VI.5: Nyquist diagram of a first order system

VI.5 Exercises

Exercise 1

Draw the polar curves of the following transfer functions:

$G(p) = \frac{1}{p+1}$ and $H(p) = \frac{1}{p(p+1)}$

If we replace the numerator by K, for what value of K this system is stable?

Solution:

a) $G(p) = \frac{1}{p+1}$, With $p = j\omega$

$$G(j\omega) = \frac{1}{j\omega + 1} = \frac{1(-j\omega + 1)}{(j\omega + 1)(-j\omega + 1)} = \frac{1 - j\omega}{\omega^2 + 1} = \frac{1}{\omega^2 + 1} - j\frac{\omega}{\omega^2 + 1}$$

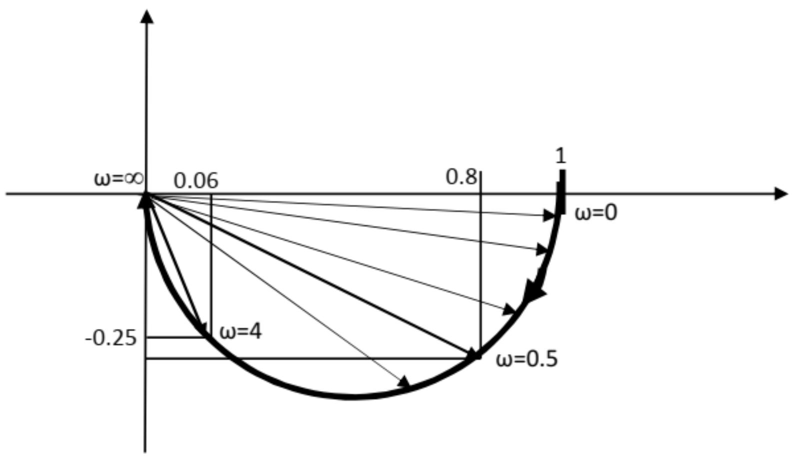
$\mathrm{Real}\big(G(j\omega)\big) = \frac{1}{\omega^2+1}$

$\mathrm{Im}\big(G(j\omega)\big) = -\frac{1}{\omega^2+1}$

$|G(j\omega)| = \frac{1}{\sqrt{\omega^2+1}}, \varphi = -arctg\omega$

$\omega \to 0 \Rightarrow \ |G(j\omega)| = 1, \varphi = 0$

$\omega \to \infty \Rightarrow \ |G(j\omega)| = 0, \varphi = \frac{\pi}{2}$



b)

$$H(p) = \frac{1}{p(p+1)}$$

$$H(j\omega) = \frac{1}{j\omega(j\omega+1)} = \frac{1}{-\omega^2 + j\omega} = \frac{-\omega^2 - j\omega}{(j\omega+1)(-\omega^2 - j\omega)} = \frac{1}{(j\omega+1)}$$

$$H(j\omega) = -j\frac{1}{(\omega^3+\omega)}$$

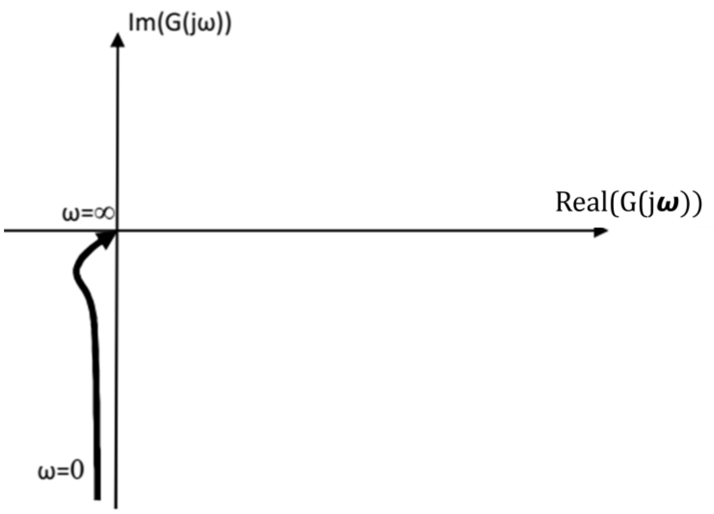
$$\text{Real}\big(G(j\omega)\big) = 0$$

$$\text{Im}\big(H(j\omega)\big) = -\frac{1}{(\omega^3+\omega)}$$

$|H(j\omega)| = \frac{1}{\sqrt{\omega^2+1}}$, $\varphi = -arctg\omega$

$\omega \to 0 \Rightarrow \; |G(j\omega)| = 1, \varphi = -\frac{\pi}{2}$

$\omega \to \infty \Rightarrow \; |G(j\omega)| = 0, \varphi = -\pi$



If: $H(p) = \frac{K}{p(p+1)}$

The value of K for this system to be stable is:

$$1 + KH_2(p) = 1 + \frac{K}{p(p+1)} = \frac{p(p+1)+K}{p(p+1)} = 0 \Rightarrow p(p+1) + K = 0$$

$$p^2 + p + K = 0$$

Using the Routh criterion:

p^2	1	K
p^1	1	0
p^0	K	

This system is stable for $K > 0$

Exercise 2

Draw the polar curves of the following transfer functions:

$F(p) = \frac{1}{p^3(p+1)}$ and $T(p) = \frac{1}{p^4(p+1)}$

Solution:

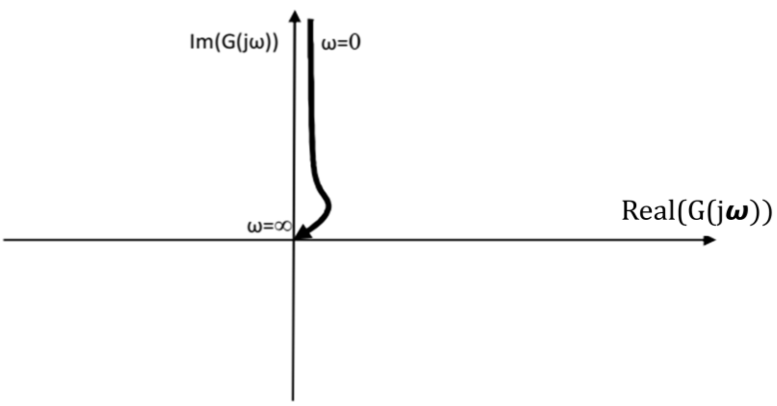
a) $F(p) = \frac{1}{p^3(p+1)}$

$F(j\omega) = \frac{1}{-j\omega^3(j\omega+1)}$

$|F(j\omega)| = \frac{1}{\omega^3\sqrt{1+\omega^2}}$, $\varphi = -\frac{3\pi}{2} - arctg\omega$

$\omega \to 0 \Rightarrow \ |F(j\omega)| = \infty, \varphi = -\frac{3\pi}{2}$

$\omega \to \infty \Rightarrow \ |F(j\omega)| = 0, \varphi = -2\pi$



b)

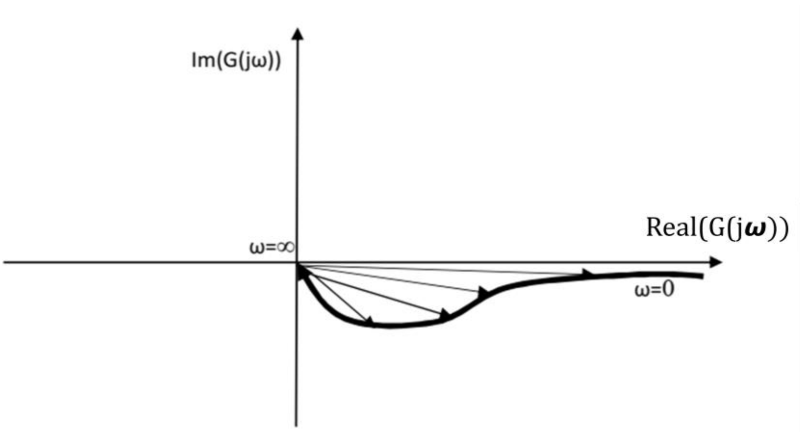
$$T(p) = \frac{1}{p^4(p+1)}$$

$$T(\,j\omega) = \frac{1}{\omega^4(j\omega+1)}$$

$$|T(j\omega)| = \frac{1}{\omega^4\sqrt{1+\omega^2}}\,,\ \varphi = -2\pi - arctg\omega$$

$$\omega \to 0 \Rightarrow \ |F(j\omega)| = \infty, \varphi = -2\pi$$

$$\omega \to \infty \Rightarrow \ |F(j\omega)| = 0, \varphi = -\frac{5\pi}{2}$$



Exercise 3

Draw the polar curve of the following systems:

$$G(p) = \frac{1}{(p+a)(p+b)}$$

$$H(p) = \frac{1}{p(p+a)(p+b)}$$

Solution:

a) $G(p) = \frac{1}{(p+a)(p+b)}$

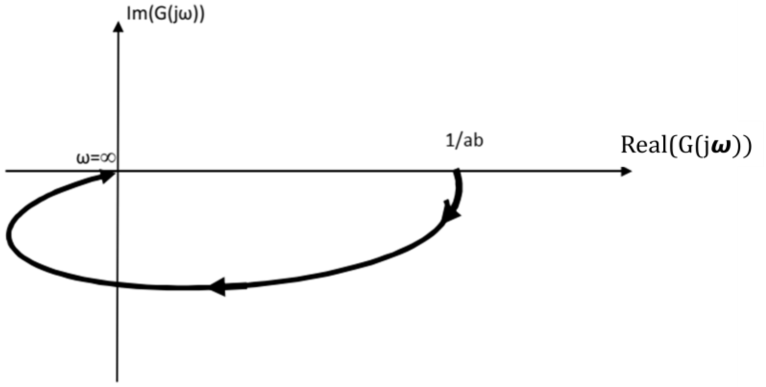
$$G(p) = \frac{1}{(j\omega+a)(j\omega+b)}$$

$$|G(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}\,\sqrt{b^2+\omega^2}}$$

$$\varphi = -arctg\frac{\omega}{a} - arctg\frac{\omega}{b}$$

Lorsque $\omega \to 0 \Rightarrow \ |G(j\omega)| = \frac{1}{ab}, \varphi = 0$

Lorsque $\omega \to \infty \Rightarrow \ |F(j\omega)| = 0, \varphi = -\pi$



b) $H(p) = \frac{1}{p(p+a)(p+b)}$

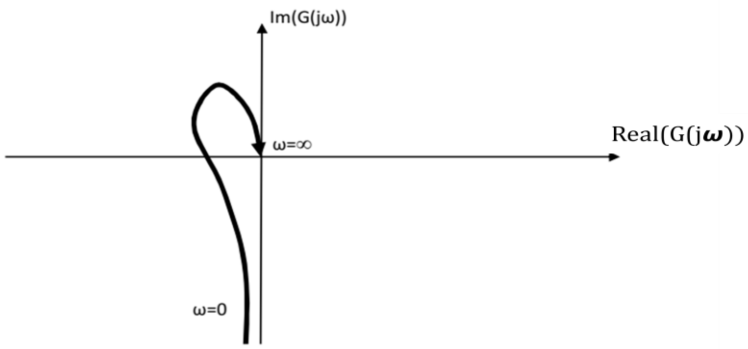
$$H(p) = \frac{1}{j\omega(j\omega+a)(j\omega+b)}$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{a^2+\omega^2}\,\sqrt{b^2+\omega^2}}$$

$$\varphi = -\frac{\pi}{2} - arctg\frac{\omega}{a} - arctg\frac{\omega}{b}$$

Lorsque $\omega \to 0 \Rightarrow \ |G(j\omega)| = \infty, \varphi = -\frac{\pi}{2}$

Lorsque $\omega \to \infty \Rightarrow \ |F(j\omega)| = 0, \varphi = -\frac{3\pi}{2}$



Exercise 4

Consider the following transfer function:

$$G(p) = \frac{1}{p-1}$$

1- Draw the polar curve of the system.

2- Study the stability of this system using the Nyquist method.

Solution:

$$G(p) = \frac{1}{p-1}$$

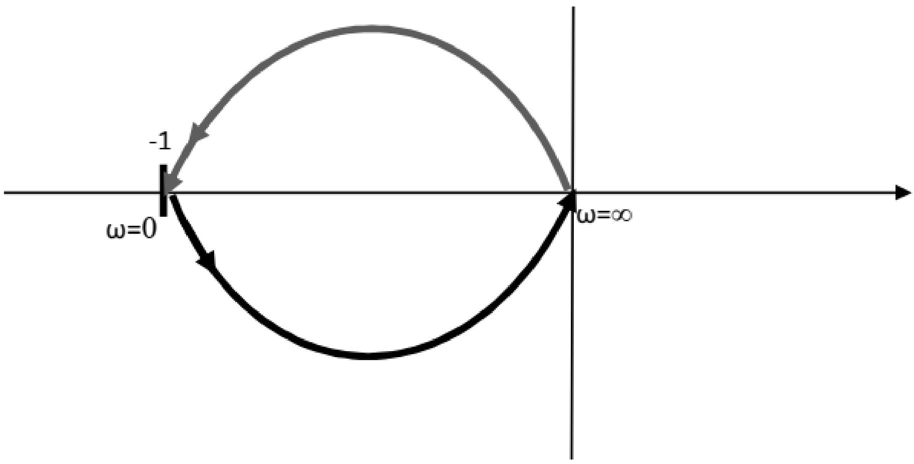
$$G(p) = \frac{1}{j\omega - 1} = \frac{1(-j\omega - 1)}{(-j\omega - 1)(j\omega - 1)} = \frac{-1 - j\omega}{\omega^2 + 1} = \frac{-1}{\omega^2 + 1} - j\frac{\omega}{\omega^2 + 1}$$

$\text{Réel}\big(G(j\omega)\big) = -\frac{1}{\omega^2+1}$, $\text{Im}\big(G(j\omega)\big) = -\frac{\omega}{\omega^2+1}$

$|G(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$, $\varphi = -arctg\omega$

$\omega \to 0 \Rightarrow \ |G(j\omega)| = 1, \varphi = 0$

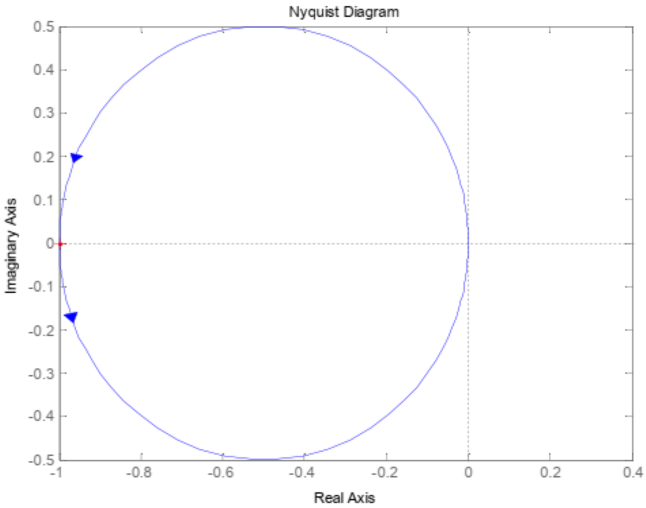
$\omega \to \infty \Rightarrow \ |G(j\omega)| = 0, \varphi = -\frac{\pi}{2}$



By applying the NYQUIST criterion, we can conclude as follows:

The open-loop transfer function G(p) admits a pole with a positive real part (1.0).

The NYQUIST exclusion contour, traversed from ω = – ∞ to ω = + ∞, surrounds (approaches) the critical point (–1.0) once, so this transfer function corresponds to a marginally stable system in closed loop.



P = 1

N = + 1

Z = P – N = 0

- No unstable pole of closed loop transfer function FTBF
- Marginally stable closed-loop system.
- This system is unstable in open loop and marginally stable in closed loop.

Exercise 5

We consider a system whose open loop transfer function is represented by the following transfer function: $H(p) = \frac{10}{p(p+2)}$

By applying the NYQUIST criterion, check the stability of this system.

Solution:

$$H(p) = \frac{10}{p(p+2)}$$

Passing into the frequency domain we obtain:

$$H(p)=\frac{10}{j\omega(j\omega+2)}=\frac{10}{j\omega(j\omega+2)}\,\frac{-j\omega(-j\omega+2)}{-j\omega(-j\omega+2)}$$

$$H(p)=\frac{-10}{(\omega^2+4)}-\frac{20j}{\omega(\omega^2+4)}$$

$$|H(j\omega)|=\frac{10}{\omega\sqrt{\omega^2+4}},\ \varphi=-\frac{\pi}{2}-arctg\,\frac{\omega}{2}$$

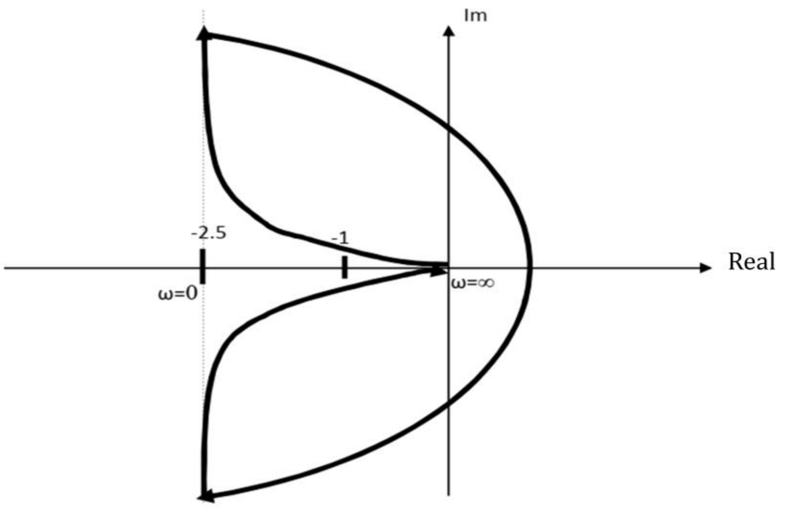
$$\omega\to 0\Rightarrow\ |G(j\omega)|=\infty,\varphi=-\frac{\pi}{2}$$

$$\omega\to\infty\Rightarrow\ |G(j\omega)|=0,\varphi=-\pi$$

$$\text{Real}\big(G(j\omega)\big)=\frac{-10}{(\omega^2+4)},\ \text{Im}\big(G(j\omega)\big)=-\frac{20j}{\omega(\omega^2+4)}$$

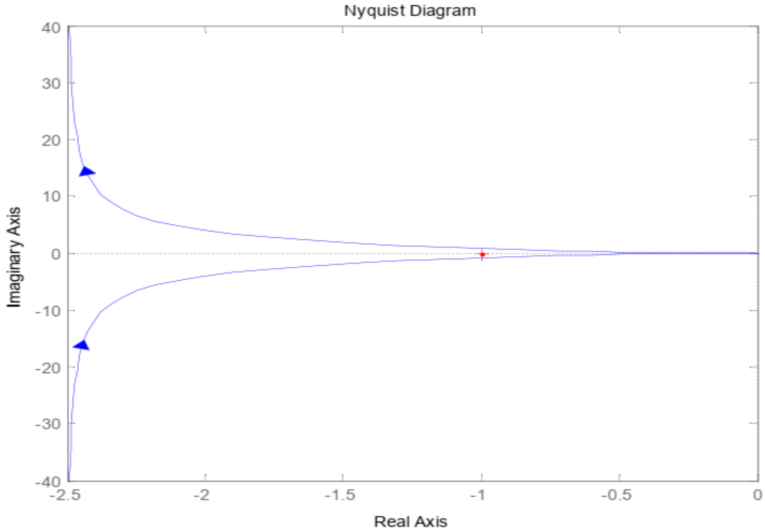
$$\omega\to 0\Rightarrow\ \text{Réel}\big(G(j\omega)\big)=-\frac{5}{2},\varphi=-\infty$$

$$\omega\to\infty\Rightarrow\ \text{Réel}\big(G(j\omega)\big)=0,\varphi=0$$



By applying the NYQUIST criterion, we can conclude that:

The transfer function G(p) does not admit any pole with a positive real part and as the NYQUIST exclusion contour does not surround the critical point (–1, 0), this transfer function corresponds to a stable loop system closed.



VI.6 Additional exercises

Exercise 1

Consider an open loop transfer function system $G(p)$ placed in a regulation loop with unitary feedback, with:

$$G(p) = \frac{K}{(p-1)(p+10)^2}$$

1-Using the Routh criterion, find the value of K, for which the closed-loop system is stable.

2- By applying the NYQUIST criterion, check the stability of this system.

Exercise 2

the system described by:

$$H(p) = \frac{25}{(0.5+0.1p)(0.5+0.2p)\,(1+2.5p)}$$

1- Drawing asymptotic and real Bode diagrams.
2- Determine graphically ω_1 , ω_π , ΔG et $\Delta\varphi$
3- Conclusion on stability

Chapter VII :

Accuracy of servo systems

VII. Accuracy of servo systems

VII.1 Introduction:

The precision of a servo, in steady state, is defined by the permanent difference $\varepsilon(t)$ which exists between the actual output and the setpoint (desired output). We have seen in the above that the role of a controlled system is to make the output s(t) follow a law determined by the input e(t). A system can be judged by its stability and also by the precision with which it follows the input law. The sources of error are both variations of the input and the effects of disturbances.

VII.1.1 Static accuracy:

It is characterized by the error at steady state when the system is subjected to canonical inputs:

- Echelon, we then speak of index error or position error.
- Ramp, we talk about dragging error or tracking or speed error.
- Parabola, we speak of error in acceleration.

VII.1.2 Dynamic precision (speed):

It is characterized by the instantaneous difference between the output and the input during the transient phase following the application of the input or after a disturbance (outside the program).

Dynamic accuracy is also called servo system quality. It measures the transient error appearing in the step response.

VII.2. Static accuracy

In general, the role of controlled systems is to make the output y(t) follow a law fixed by a setpoint x(t), with the ideal case $\varepsilon(t) = x(t) - y(t) = 0$.

In practice we have:

-The input varies: The system operates as a follower and performs the servo function.

-Constant input: but a disturbance signal can be superimposed on the useful signal at any point in the chain**.**

We have: $\varepsilon(p) = X(p) - R(p)$

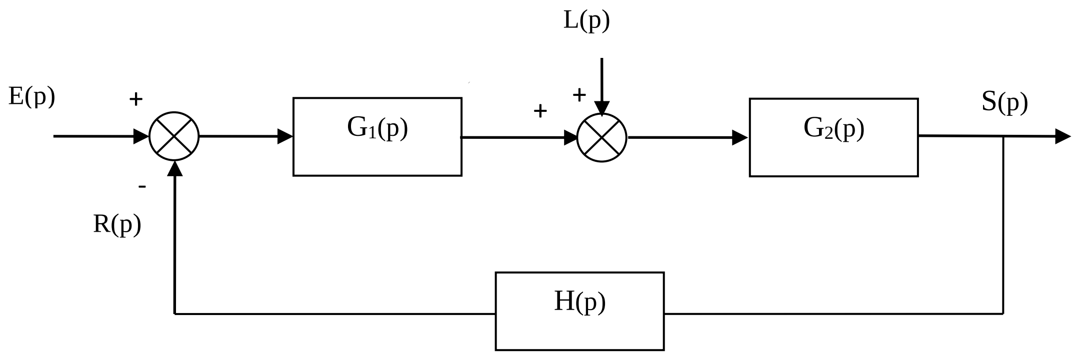


Figure VII.1: Slave system

$R(p) = \varepsilon(p). G_1(p). G_2(p). H(p) + P(p). G_2(p). H(p)$

And $R(p) = G_1(p). G_2(p). H(p) = T(p)$ closed loop transfer function

So: $\varepsilon(p) = E(p) - [\varepsilon(p). G_1(p). G_2(p). H(p) + P(p). G_2(p). H(p)]$

$\Rightarrow \varepsilon(p) = \frac{1}{1+T(p)}. E(p) - \frac{G_2(p).H(p)}{1+T(p)}. P(p) = \varepsilon_e(p) - \varepsilon_{pre}(p)$

The error ε(p) at each instant is the sum of two errors:

- Error due to input variations: $\varepsilon_e(p) = \frac{1}{1+T(p)}$
- Error due to disturbance: $\varepsilon_{pre}(p) = \frac{G_2(p).H(p)}{1+T(p)}. P(p)$

By virtue of the superposition theorem we have $\varepsilon(t) = \varepsilon_e(t) - \varepsilon_{pre}(t)$

In general, we distinguish:

The quality of a controlled system is judged by its stability and speed, but also by the precision with which it follows the input law. The sources of error are both the variations of the input but also the effects of disturbances. We distinguish two (2) types of errors: Static precision (or error in steady state): This is the instantaneous difference between the output and the setpoint during the transient phase following a setpoint variation or following a disturbance. While it is normal for such an error to exist; we nevertheless want it to be as small as possible and to last as short a time as possible.

- Static precision (or error in steady state): This is the instantaneous difference between the output and the setpoint during the transient phase following a setpoint variation or following a disturbance. While it is normal for such an error to exist, we nevertheless want it to be as small as possible and to last as short a time as possible.

$$\varepsilon(\infty) = \lim_{t\to\infty} \varepsilon(t)$$

- Dynamic pressure (transient error): This is the steady-state error between the output and the input law. $\varepsilon(t) = S_d(t) - S(t)$.

In what follows, we are interested in the permanent error also called static error.

By applying the final value theorem we will have: $\varepsilon_s(\infty) = \lim_{t\to\infty} \varepsilon(t) = \lim_{t\to\infty} p\varepsilon(p)$.

For a class 1 system and above the position error is zero. This means that such systems are characterized by perfect static precision.

VII.3 System without disturbance and with variable input

We have: $P(p) = 0$ if we put $H(p) = 1$ and $G_1(p).G_2(p) = G(p)$.

$$\varepsilon_e(p) = E(p) - S(p) = E(p) - G(p).\varepsilon_e(p)$$

$$\varepsilon_e(p) = \frac{E(p)}{1 + G(p)} = \frac{E(p)}{1 + T(P)}$$

Where $T(p) = G(p)$

Therefore the error $\varepsilon_e(p)$is linked on the one hand to the shape of the input signal $E(p)$ and on the other hand to the shape of the open loop transfer function (F.T.B.O).

VII.3.1 Influence of entry

- If the input is a step, the error is called position error and is denoted ε_p.
- If the input is a ramp, the error is called speed or lag error, denoted ε_v.
- If the input is a parabola the error is called acceleration error and is denoted ε_a.

VII.3.2 Influence of the F.T.B.O

To calculate the error ε, it is wise to put the F.T.B.O in the following form:

$$G(p) = \frac{K}{P^{\alpha}} \frac{\prod_{i=1}^{m}(p - Z_i)}{\prod_{i=1}^{n}(p - p_j)} = \frac{K}{P^{\alpha}} \frac{N(p)}{D(p)}$$

With $H(p) = 1$

$N(0) = D(0) = 1$

$K = \frac{\prod -Z_i}{\prod -p_j}$: Statistical gain of the open loop system

α is called the type of the system (or number of integrations)

VII.3.2.1 Type 0 system (no integration) ⇒ α=0

So we have: $F.T.B.O = K.\frac{N(p)}{D(p)}$

$$\varepsilon(p) = \frac{E(p)}{1 + F.T.B.O} = \frac{E(p)}{1 + K.\frac{N(p)}{D(p)}} = \frac{D(p)}{D(p) + KN(p)} E(p)$$

a) Entry into echelon

An amplitude echelon $A \Rightarrow E(p) = \frac{A}{p}$

$\Rightarrow \varepsilon(\infty) = \lim_{p \to 0} p\varepsilon(p) = \lim_{p \to 0} p.\frac{A}{p} \frac{1}{\left(1+K.\frac{N(p)}{D(p)}\right)} = \frac{A}{1+K_p}$

$K_p = \lim_{p \to 0} K \frac{N(p)}{D(p)} = K$ donc $\varepsilon_p(\infty) = \frac{A}{1+K} = cste$

K_p is called position constant.

b) Ramp entry

$E(t) = A.t \Rightarrow E(p) = \frac{A}{p^2}$ Où $A = cst$

$\Rightarrow \varepsilon_v(\infty) = \lim_{p \to 0} p\varepsilon(p) = \lim_{p \to 0} p.\frac{A}{p^2} \frac{1}{\left(1+K.\frac{N(p)}{D(p)}\right)} \to \infty \Rightarrow \varepsilon_v(\infty) = \frac{A}{K_v} \to \infty$

$$K_v = \lim_{p\to 0} K.p.\frac{N(p)}{D(p)} = 0$$

K_v : is called the rate constant.

c) Entry into parabola

$E(t) = \frac{1}{2}A.t^2 \Rightarrow E(p) = \frac{A}{p^3}$

$$\Rightarrow \varepsilon_a(\infty) = \lim_{p\to 0} p\varepsilon(p) = \lim_{p\to 0} p.\frac{A}{p^3}\frac{1}{\left(1+K.\frac{N(p)}{D(p)}\right)} \to \infty \Rightarrow \varepsilon_a(\infty) \to \infty$$

$$K_a = \lim_{p\to 0} K.p^2.\frac{N(p)}{D(p)} = 0\,, \varepsilon_a(\infty) = \frac{A}{K_a}$$

K_a : is called the acceleration constant.

A zero-type system does not follow either a velocity input or an acceleration input.

VII.3.2.2 Type 1 system (an integration) ⇒ α=1

The F.T.B.O is now written in the form: $F.T.B.O = K.\frac{N(p)}{p.D(p)}$

$$\varepsilon(p) = \frac{E(p)}{1 + F.T.B.O} = \frac{E(p)}{1 + K.\frac{N(p)}{p.D(p)}}$$

a) Entry into echelon

$$\varepsilon(\infty) = \lim_{p\to 0} p\varepsilon(p) = \lim_{p\to 0} p.\frac{A}{p^2}\frac{1}{\left(1+K.\frac{N(p)}{D(p)}\right)} = 0,\ \varepsilon_p(\infty) = 0$$

$$K_p = \lim_{p\to 0}\frac{K}{p}\frac{N(p)}{D(p)} = \infty$$

b) Ramp entry

$$\varepsilon_v(\infty) = \lim_{p\to 0} p.\frac{A}{p^2}\frac{1}{\left(1 + K.\frac{N(p)}{D(p)}\right)} \to \infty \Rightarrow \varepsilon_v(\infty) = \frac{A}{K_v} \to \infty$$

$$K_v = \lim_{p\to 0}\frac{K}{p}.p.\frac{N(p)}{D(p)} = K$$

c) Entry into parabola

$$\varepsilon_a(\infty) = \lim_{p\to 0} p.\frac{A}{p^3}\frac{1}{\left(1 + K.\frac{N(p)}{D(p)}\right)} \to \infty \Rightarrow \varepsilon_a(\infty) \to \infty$$

$$K_a = \lim_{p\to 0} \frac{K}{p}.p^2.\frac{N(p)}{D(p)} = 0$$

An integrator cancels the position error and makes the trailing error finite

VII.3.2.3 Type 2 system (two integrator) a = 2

The F.T.B.O is now written in the form:

$$F.T.B.O = \frac{K}{p^2}.\frac{N(p)}{D(p)} \Rightarrow \varepsilon(p) = \frac{E(p)}{\left(1 + \frac{K}{p^2}.\frac{N(p)}{D(p)}\right)}$$

a) Entry into echelon

$$\varepsilon_p(\infty) = \lim_{p\to 0} p\varepsilon(p) = \lim_{p\to 0} p.\frac{A}{p}\frac{1}{\left(1+\frac{K}{p^2}.\frac{N(p)}{D(p)}\right)} = 0,\ \varepsilon_p(\infty) = 0$$

$$K_p = \lim_{p\to 0} \frac{K}{p^2}\frac{N(p)}{D(p)} \to \infty$$

b) Ramp entry

$$\varepsilon_v(\infty) = \lim_{p\to 0} p.\frac{A}{p^2}\frac{1}{\left(1 + \frac{K}{p^2}.\frac{N(p)}{D(p)}\right)} = 0 \Rightarrow \varepsilon_v(\infty) = 0$$

$$K_v = \lim_{p\to 0} \frac{K}{p^2}.p.\frac{N(p)}{D(p)} = K$$

c) Entry into parabola

$$\varepsilon_a(\infty) = \lim_{p\to 0} p.\frac{A}{p^3}\frac{1}{\left(1 + \frac{K}{p^2}.\frac{N(p)}{D(p)}\right)} = \frac{A}{K} \Rightarrow \varepsilon_a(\infty) = \frac{A}{K}$$

$$K_a = \lim_{p\to 0} \frac{K}{p^2}.p^2.\frac{N(p)}{D(p)} = K$$

In a type two system, the position and drag errors are zero and the acceleration error becomes finite.

It can be seen that if $a > 2 \Rightarrow K_a = \lim_{p \to 0} \frac{K}{p^a} . p^2 . \frac{N(p)}{D(p)} \to \infty$, $\varepsilon_a(\infty) = 0$

The above can be summarized in the table below:

Precision static	Type 0 system $\alpha = 0$	Type 1 system $\alpha = 1$	Type 2 system $\alpha = 2$	Type 3 system $\alpha = 3$
Position error (ε_p)	$\frac{A}{1 + K_p}$	0	0	0
Training Error (ε_v)	$+\infty$	$\frac{A}{K_v}$	0	0
Acceleration error(ε_a)	$+\infty$	$+\infty$	$\frac{A}{K_a}$	0

We should not quickly deduce from the table that it is enough to add an integration for the system to be precise, in fact each integration also adds a phase shift of -90°, the system risks becoming unstable, hence a precision-stability dilemma .

VII.4 Accuracy with respect to disturbances

A disturbance is an additional input to the system that cannot be controlled. These disturbances have an influence on the servo-control. Here we want to evaluate this influence quantitatively. (A good servo should make this influence minimal).

In the example of Figure VII.2, the effect of the disturbance on the output can be calculated by studying the block diagram considering that the input E(p) = 0.

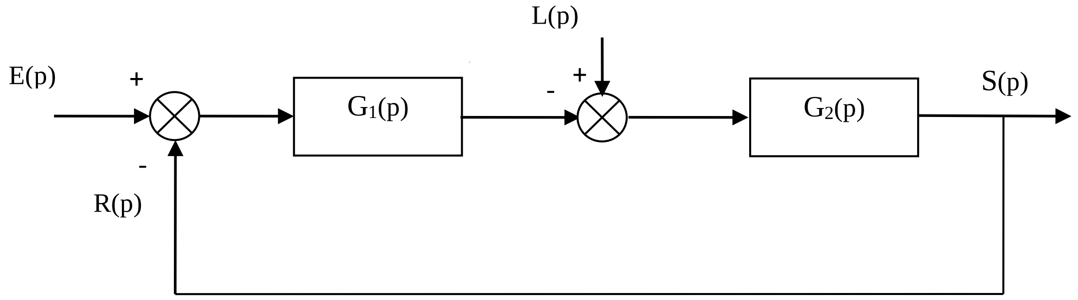


Figure VII.2: block diagram of a servo with a disturbance

Either

$$G_1(p) = \frac{K_1}{p^{\alpha_1}}.\frac{N_1(p)}{D_1(p)}$$

$$G_2(p) = \frac{K_2}{p^{\alpha_2}}.\frac{N_2(p)}{D_2(p)}$$

With $N_i(0) = 1$ and $D_i(0) = 1$

In this case, the gap $\varepsilon(p)$ is the superposition of two gaps: $\varepsilon(p) = \varepsilon_e(p) - \varepsilon_{pre}(p)$

- $\varepsilon_e(p) = \varepsilon(p)|_{L(p)=0} = \frac{E(p)}{1+G_1(p)G_2(p)}$ which is due to the input $E(p)$ such that $L(p) = 0$;
- $\varepsilon_{per}(p) = \varepsilon(p)|_{E(p)=0} = \frac{G_2(p)}{1+G_1(p)G_2(p)}.L(p)$ which is due to the perturbation $L(p)$ such that $E(p) = 0$.

These two differences can be found by directly reading the block diagram:

We have

$$\varepsilon(p) = E(p) - S(p) = E(p) - (G_1(p)\varepsilon(p) - L(p)).G_2(p)$$

$$\Rightarrow \varepsilon(p)\big(1 + G_1(p)G_2(p)\big) = E(p) + L(p).G_2(p)$$

$$\Rightarrow \varepsilon(p) = \frac{E(p)}{1 + G_1(p)G_2(p)} + \frac{G_2(p)}{1 + G_1(p)G_2(p)}.L(p) = \varepsilon_e(p) + \varepsilon_{per}(p)$$

We deduce that the static error is the superposition of two errors: $\varepsilon_\infty = \varepsilon_{e\infty}(p) + \varepsilon_{per\infty}(p)$

- $\varepsilon_{e\infty} = \lim\limits_{p\to 0} p.\varepsilon_e(p) = \lim\limits_{p\to 0} p.\frac{E(p)}{1+G_1(p)G_2(p)}$ which is due to the input $E(p)$.
- $\varepsilon_{per\infty} = \lim\limits_{p\to 0} p.\varepsilon_{per}(p) = \lim\limits_{p\to 0} p.\frac{G_2(p)L(p)}{1+G_1(p)G_2(p)}$ which is due to the disturbance $L(p)$.

The error $\varepsilon_{e\infty}$ due to the input $E(p)$ has already been studied in the previous paragraph.

Let us determine the error ε_{per} due to a constant disturbance: $L(t) \to L_0.u(t) \Rightarrow L(p) = \frac{L_0}{p}$,

$$\varepsilon_{per\infty} = \lim_{p\to 0} p.\varepsilon_{per}(p) = \lim_{p\to 0} p.\frac{G_2(p)L(p)}{1 + G_1(p)G_2(p)} = \lim_{p\to 0} p.\frac{G_2(p)}{1 + G_1(p)G_2(p)}.\frac{L_0}{p}$$

$$\varepsilon_{per\infty} = \lim_{p \to 0} \frac{K_2 p^{\alpha_1}}{p^{\alpha_1+\alpha_2} + K_1 K_2} . L_0$$

- If $\alpha_1 = 0$ (no integration in $G_1(p)$) so $\varepsilon_{per\infty}$ is not zero: $\varepsilon_{per\infty} = \lim_{p \to 0} \frac{K_2}{p^{\alpha_2}+K_1K_2} . L_0$
- If $\alpha_1 \geq 0$ (at least one integration in$G_1(p)$) so $\varepsilon_{per\infty} = 0$

VII.5 Exercises:

Exercise 1

Consider the system in Figure VII.1 initially subjected to a unit step input, then to a unit ramp input. Then determine the steady-state errors.

Solution:

a) Entry into unit echelon

We have $X(p) = \frac{1}{p}$ and FTBO is worth $KG(p) = \frac{k}{p^3+5p^2+2p+1}$. Therefore, the Laplace transform of the error is:

$$\varepsilon(p) = \frac{X(p)}{1 + KG(p)} = \frac{p^3 + 5p^2 + 2p + 1}{p(p^3 + 5p^2 + 2p + 1 + k)}$$

$$\varepsilon_p = \lim_{p \to 0} p . \varepsilon(p) = \frac{1}{1+k}$$

If $k = 0{,}5$ We can note that $\varepsilon(\infty) = \frac{2}{3}$ either 66%.

b) Ramp entry :

We have $X(p) = \frac{1}{p^2}$ and FTBO is worth $KG(p) = \frac{k}{p^3+5p^2+2p+1}$.

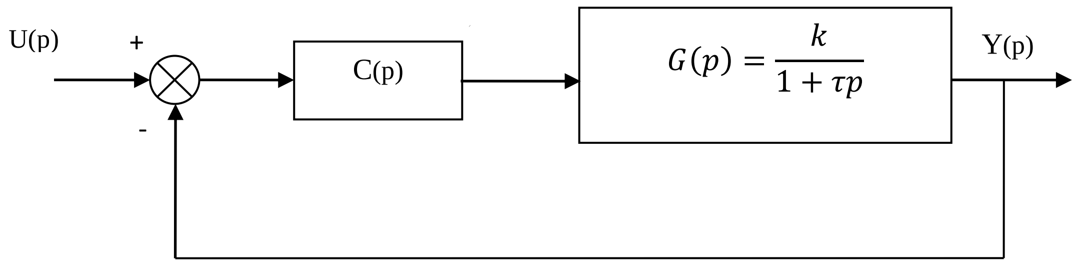
Therefore, the Laplace transform of the error is :

$$\varepsilon(p) = \frac{X(p)}{1 + KG(p)} = \frac{p^3 + 5p^2 + 2p + 1}{p^2(p^3 + 5p^2 + 2p + 1 + k)}$$

$$\varepsilon_p = \lim_{p \to 0} p . \varepsilon(p) = \infty$$

Exercise 2

Let the first order system be given by: $G(p) = \frac{K}{1+\tau p}$



1- Give the expression for the static error, when we use a proportional regulator of the gain K_p for an index response and what is your remark.

Solution:

The functional diagram of the closed loop system is given by:

The closed loop transfer function is:

$$H(p) = \frac{C(p)G(p)}{1 + C(p)G(p)} = \frac{K_p \frac{k}{1+\tau p}}{1 + K_p \frac{k}{1+\tau p}} = \frac{K_p k}{1 + \tau p + K_p k}$$

The static error is given by:

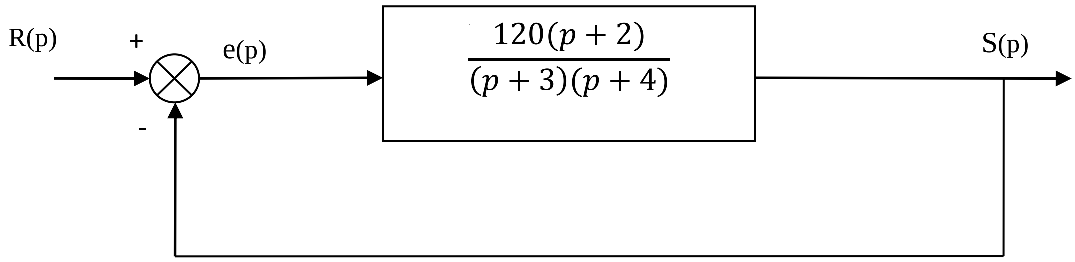
$$\varepsilon_s(\infty) = \lim_{y\to 0} \varepsilon(t) = \lim_{t\to 0} \mathrm{E(p)} = \lim_{t\to 0} p[U(p) - Y(p)] = \lim_{t\to 0} p[\frac{1}{p} - \frac{1}{p}(\frac{K_p k}{1 + \tau p + K_p k})]$$

$$\varepsilon_s = 1 + K_p \frac{k}{1+\tau p} = \frac{1}{1 + K_p k}$$

We notice that the static error ε_s is inversely proportional to the value of K_p but ε_s always remains different from zero.

Exercise 3

Find the steady-state errors for inputs of $5.u(t)$, $5.tu(t)$, and $5.t^2u(t)$ to the system shown in Figure below. The function *u*(*t*) is the unit step.



Solution:

First we verify that the closed-loop system is indeed stable. For this example we leave out the details. Next, for the input 5u(t), whose Laplace transform is 5=s, the steady-state error will be five times as large as that given by the equation:

$$e(\infty) = e_{step}(\infty) = \lim_{p\to 0} \frac{p(\frac{1}{p})}{1+G(p)} = \frac{1}{1+\lim_{p\to 0} G(p)}$$

Or

$$e(\infty) = e_{step}(\infty) = \lim_{p\to 0} \frac{5}{1+G(p)} = \frac{5}{1+20} = \frac{5}{21}$$

For the input $5.\,tu(t)$, whose Laplace transform is $5/p^2$, the steady-state error will be five times as large as that given by the equation:

$$e(\infty) = e_{ramp}(\infty) = \lim_{p\to 0} \frac{p(\frac{1}{p^2})}{1+G(p)} = \lim_{p\to 0} \frac{1}{p+p\,G(p)} = \frac{1}{\lim_{p\to 0} pG(p)}$$

Or:

$$e(\infty) = e_{ramp}(\infty) = \lim_{p\to 0} \frac{5}{p\,G(p)} = \frac{5}{0} = \infty$$

For the input $5.\,t^2u(t)$, whose Laplace transform is $10/s^3$, the steady-state error will be 10 times as large as that given by the equation:

$$e(\infty) = e_{parabola}(\infty) = \lim_{p\to 0} \frac{p(\frac{1}{p^3})}{1+G(p)} = \lim_{p\to 0} \frac{1}{p^2+p^2\,G(p)} = \frac{1}{\lim_{p\to 0} p^2G(p)}$$

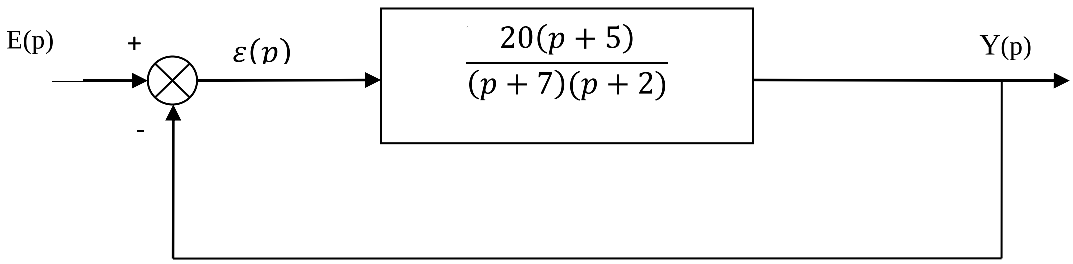
Or:

$$e(\infty) = e_{parabola}(\infty) = \frac{1}{\lim_{p\to 0} p^2G(p)} = \frac{10}{0} = \infty$$

VII.5 Additional exercises

Exercise 1

Consider the following system:

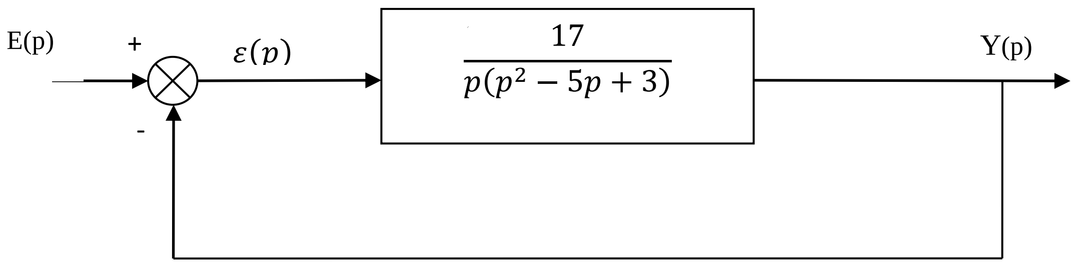


1- Study of the stability of the following system:

2- Calculate the static error due to a unit step input for the system

Exercise 2

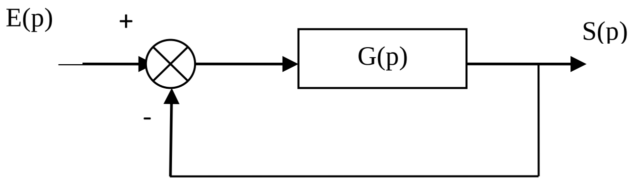
Consider the following system:



1- Calculate the static error due to a ramp input.

Exercise 3

Consider the following slave system:



We give the transfer function in Closed-Loop BF

$$F(p) = \frac{K}{Mp^2 + Np + K}$$

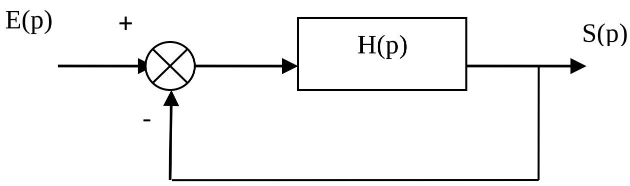
1- What is the class of this system?

2- If the input to the system is a ramp with slope 1, calculate the error based on K, M and N. Comment on your results?

3- Confirm your results using error constants?

Exercise 4

A unitary feedback servo system and its open loop transfer function $H(p)$ ($H(p)$ is arbitrary).



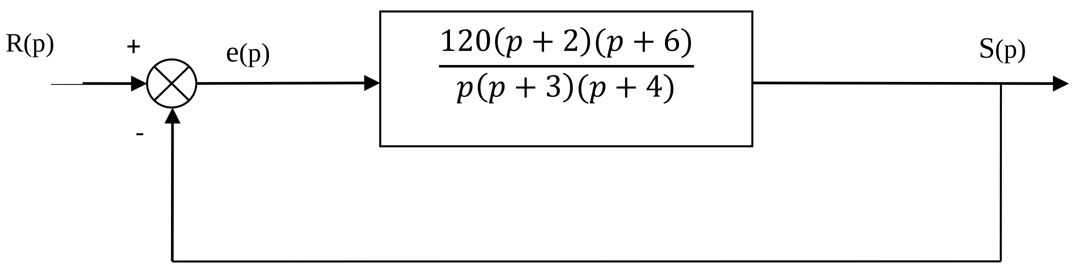
We apply to the input of this S.A a command of the form $e(t) = \alpha + \beta t + \frac{\gamma}{2} t^2$

1- Give the expression of the error as a function of $H(p)$ and the error constants, when t→∞.

2- What is the class of the system so that the error is $\varepsilon \neq \infty$.

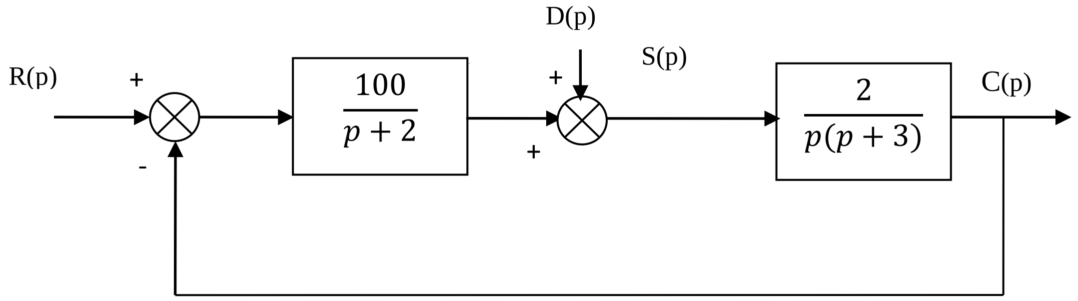
Exercise 5

Find the steady-state errors for inputs of $5.u(t)$, $5.tu(t)$, and $5.t^2u(t)$ to the system shown in Figure below. The function $u(t)$ is the unit step.



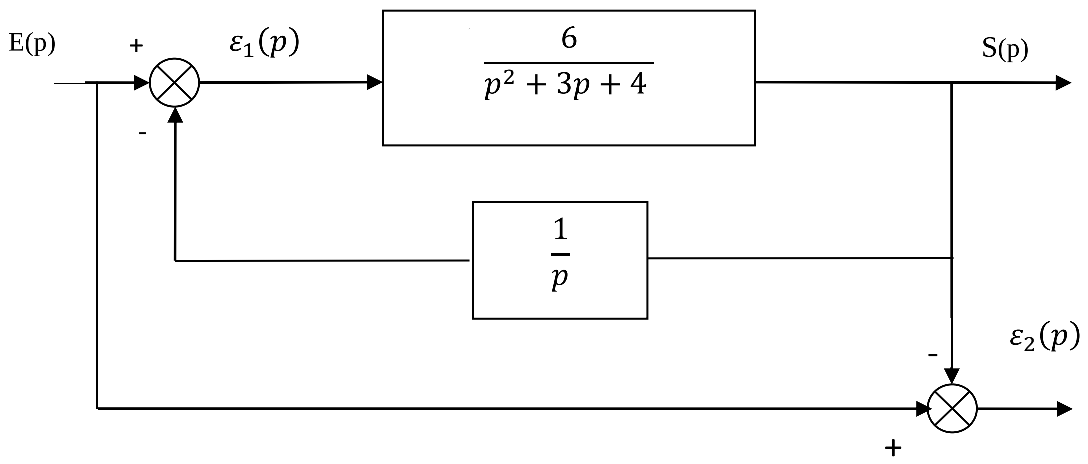
Exercise 6

Find the total steady-state error due to a unit-step input and a unit-step disturbance in the system in the figure below.



Exercise 7

Perform the static study of the errors $\varepsilon_1(p)$ and $\varepsilon_2(p)$ for the step, ramp and acceleration type inputs.



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