## General Introduction

Most people know, at least in some vague way, that the sophisticated technology that drives our society has been driven in turn by fundamental discoveries of physics. But, just what is physics? It derives its present name from the Greek word for nature; it was previously called natural philosophy.

Physics can be defined as the science that deals with matter, energy, motion and force. It studies the fundamental building blocks of the universe and how they interact. It seeks answers to such fundamental questions as: What kind of world do we live in? How does it work? What are the fundamental laws of nature? Thus, physics is the basic science from which all others have derived.

Transistors, microchips, lasers, computers, telecommunications, nuclear power and space travel are among the many applications of physics that are so pervasive in our times. In our daily newspaper or weekly magazine, we often find articles that attempt to explain to a lay public a variety of topics related to physics. These might be sophisticated experiments on fundamental particles of matter; space probes and their missions; discoveries of astronomy in very remote regions of space; exotic new theories on the nature of matter, or the universe as a whole.

The relevance of physics is all around us. Although not as palpable as in the days of the Cold War with the Soviet Union, the terrifying threat of nuclear holocaust still hangs over all mankind. With so many programs competing for federal funds, government support of very expensive scientific ventures has become an issue of public interest. Except for fundamentalist groups, few, if any, religious leaders dare challenge the experimental findings of physics. No metaphysical speculation about the nature of reality1, whether by lay people or professional philosophers, can ignore these findings. We clearly live in times that require at least some modest level of literacy in physics, one of the most profound achievements of the human mind. Unfortunately, physics is the least known and the most intimidating of all sciences. This is true even for many who are literate at some level about other human endeavors.

Among the factors that make physics appear so alien to so many people are the difficulty of many of its concepts, its pervasive use of advanced mathematics and cryptic symbolism, and the sophistication of its instruments, whose complexity goes far beyond the telescope first used by Galileo in 1609 .

Although strongly intimidated by physics, much of the lay public has been, and still is, intrigued by the fundamental nature of its inquiry. This is shown by the success of dozens of books that have been written since Stephen Hawking's "A Brief History of Time" (1988) became a best seller. In most of the popular books on the market, however, the bulk of the material is at a level of presentation and detail that goes beyond the background and interest of much of the general public. (A notable exception is Roger S. Jones' very readable "Physics for the Rest of Us", Contemporary Books, 1992). Many of these books focus on specific areas of scientific endeavor; some are offered as part of a series that covers a broader area of physics.

In these chapters, we will begin to explore physics and leading up through a review of Sir Isaac Newton and the laws of physics that bear his name. We will also be introduced to the standards scientists use when they study physical quantities and the interrelated system of measurements most of the scientific community uses to communicate in a single mathematical language. Study the limits of our ability to be accurate and precise, and the reasons scientists go to painstaking lengths to be as clear as possible regarding their own limitations. Finally, we will study extensions of the application of Newton's laws in everyday life.

## List of Figures

## List of Figures

Figure 1: Speed vs. velocity ..... 12
Figure 2: Direction of motion ..... 13
Figure 3: A hammer and a feather falling in air and in a vaccum ..... 16
Figure 4: Distance illustration. ..... 20
Figure 5: Independence of motion ..... 21
Figure 6: Projectile motion ..... 22
Figure 7: Force exerted by a stretched spring ..... 27
Figure 8: Different forces exerted on the same mass ..... 29
Figure 9: Frictional forces ..... 36
Figure 10: Actions of the same force ..... 38
Figure 11: Tension ..... 39
Figure 12: Shearing forces ..... 40
Figure 13: An inward force on all surfaces ..... 41
Figure 14: Plastic ruler oscillations ..... 47
Figure 15: Ruler equilibrium position ..... 47
Figure 16: Guitar strings vibrating ..... 49
Figure 17: An uncomplicated simple harmonic oscillator ..... 50
Figure 18: Simple pendulum ..... 51
Figure 19: Example of uniform circular motion ..... 53
Figure 20: Simple harmonic motion ..... 53
Figure 21: A harmonic oscillator with a small amount of damping ..... 54
Figure 22: An example of an idealized ocean wave ..... 55
Figure 23: An example of a transverse wave ..... 56
Figure 24: An example of a longitudinal wave ..... 56
Figure 25: Transverse and longitudinal waves ..... 57

## List of tables

Table 1: SI units ...................................................................................................................... 4
Table 2: Derived units .5

Table 3: standard SI prefixes................................................................................................... 6
Table 4: Properties of the Four Basic Forces ......................................................................... 31

## Table of Content

General Introduction .....  i
List of Figures ..... iii
List of tables ..... iv
Table of Content ..... v
Introduction ..... 1

1. Measurements ..... 3
1.1 Introduction ..... 3
1.2 Units and Systems: ..... 3
1.2.1 International System of Units (SI): ..... 4
1.2.2 Derived units: ..... 4
1.3 Expressing larger and smaller physical quantities ..... 6
2 One-dimensional Kinematics ..... 11
2.1 Displacement ..... 11
2.1.1 Position ..... 11
2.1.2 Distance ..... 12
2.2 Vectors and Scalars Coordinate Systems ..... 12
2.3 Coordinate Systems of one-dimensional motion. ..... 13
2.4 Time, velocity and speed ..... 13
2.4.1 Time. ..... 13
2.4.2 Velocity ..... 14
2.4.3 Speed ..... 14
2.5 Acceleration ..... 15
2.6 Falling Objects ..... 15
2.6.1 Gravity ..... 15
3 Two dimension Kinematics ..... 20
3.1 Vector Addition and Subtraction: Graphical Methods ..... 21
3.2 Vector Addition and Subtraction: Analytical Methods ..... 21
3.3 Projectile Motion ..... 22
4 Introduction to Dynamics: Newton's Laws of Motion ..... 27
4.1 Development of Force Concept. ..... 27
4.2 Newton's First Law of Motion: Inertia ..... 28
4.3 Newton's Second Law of Motion: Concept of a System ..... 28
4.4 Newton's Third Law of Motion: Symmetry in Forces ..... 30
4.5 Further Applications of Newton's Laws of Motion ..... 30
4.6 Extended Topic: The Four Basic Forces ..... 30
5 Further Applications of Newton's Laws ..... 36
5.1 Friction ..... 36
5.2 Drag forces ..... 37
5.3 Elasticity: Stress and Strain ..... 37
5.4 Changes in Length-Tension and Compression: Elastic Modulus ..... 38
5.5 Sideways Stress: Shear Modulus ..... 40
5.6 Changes in Volume: Bulk Modulus ..... 40
6 Introduction to oscillatory systems and waves ..... 46
6.1 Hooke's Law: Stress and Strain Revisited ..... 46
6.2 Energy in Hooke's Law of Deformation ..... 48
6.3 Period and Frequency in Oscillations ..... 48
6.4 Simple Harmonic Motion: A Special Periodic Motion ..... 49
6.5 The Simple Pendulum ..... 51
6.6 Uniform Circular Motion and Simple Harmonic Motion ..... 52
6.7 Damped Harmonic Motion. ..... 54
6.8 Forced Oscillations and Resonance ..... 54
6.9 Waves ..... 54
6.9.1 Transverse and Longitudinal Waves ..... 56
Sources and References ..... 64

## Introduction

This manuscript is meant for L1 (freshmen) students, it settles the basics of Physics and aims at enriching learners' vocabulary through glossary and consolidates their knowledge through different tasks and activities.

The final objective of this manuscript is to enable learners to see and notice that Physics is everywhere. Its applications range from driving a car to launching a rocket, from a skater whirling on ice to a neutron star spinning in space, and from taking your temperature to taking a chest X-ray.

This work may also be regarded as a source book for teachers when preparing lectures or when dealing with some topics as these latters are introduced conceptually with a steady progression to precise definitions, illustrations through figures and examples and finally with activities that settle learners' understanding.

Organized in six chapters with glossaries and activities, this document helps freshmen students to settle their previous knowledge and contributes to enrich their technical vocabulary.

## I. Measurements



1. Measurements
1.1 Introduction
1.2 Units and Systems

### 1.2.1 International System of Units (SI)

### 1.2.2 Derived units

1.3 Expressing larger and smaller physical quantities

Check your understanding
Glossary

## Chapter I: Measurements

## 1. Measurements

### 1.1 Introduction :

Measurement is the quantification of attributes of an object or event, which can be used to compare with other objects or events. The scope and application of measurement are dependent on the context and discipline.

The measurement of a property may be categorized by the following criteria: type, magnitude, unit, and uncertainty. They enable unambiguous comparisons between measurements.

- The level of measurement is a taxonomy for the methodological character of a comparison. For example, two states of a property may be compared by ratio, difference, or ordinal preference. The type is commonly not explicitly expressed, but implicit in the definition of a measurement procedure.
- The magnitude is the numerical value of the characterization, usually obtained with a suitably chosen measuring instrument.
- A unit assigns a mathematical weighting factor to the magnitude that is derived as a ratio to the property of an artifact used as standard or a natural physical quantity.
- An uncertainty represents the random and systemic errors of the measurement procedure; it indicates a confidence level in the measurement. Errors are evaluated by methodically repeating measurements and considering the accuracy and precision of the measuring instrument.

Measurements most commonly use the International System of Units (SI) as a comparison framework. The system defines seven fundamental units: kilogram, meter, candela, second, ampere, kelvin, and mole. The first proposal to tie an SI base unit to an experimental standard was by Charles Sanders Peirce (1839-1914), who proposed to define the meter in terms of the wavelength of a spectral line.

### 1.2 Units and Systems:

Before SI units were widely adopted around the world, the British systems of English units and later imperial units were used in Britain, the Commonwealth and the United States. The system came to be known as U.S. customary units in the United States and is still in use there and in a few Caribbean countries. These various systems of measurement have at

## Chapter I: Measurements

times been called foot-pound-second systems after the Imperial units for length, weight and time.

The metric system on the other hand is a decimal system of measurement based on its units for length, the meter and for mass, the kilogram. We are more interested in this textbook with SI units and derived units.

### 1.2.1 International System of Units (SI):

The International System of Units (abbreviated as SI from the French language name Système International d'Unités) is the modern revision of the metric system. It is the world's most widely used system of units. The SI was developed in 1960 from the meter-kilogramsecond (MKS) system, rather than the centimeter-gram-second (CGS) system, which, in turn, had many variants. SI units are expressed in tab.1.1 as such:

| Base quantity | Base unit | Symbol | Defining constant |
| :--- | :--- | :--- | :--- |
| Time | second | s | hyperfine splitting in caesium-133 |
| Length | meter | m | Speed of light, $c$ |
| Mass | kilogram | kg | Planck constant, $h$ |
| Electric Current | ampere | A | Elementary charge, $e$ |
| Temperature | kelvin | K | Boltzmann constant, $k$ |
| Amount <br> substance of | mole | mol | Avogadro constant $N_{\mathrm{A}}$ |
| Luminous Intensity | candela | cd | Luminous efficacy of a <br> source $K_{\mathrm{cd}}$ |

Table 1: SI units

### 1.2.2 Derived units:

Derived units can be expressed in terms of products or quotients of base units. Derived units are displayed in tab.1.2.

## Chapter I: Measurements

| Derived Quantities |  | Equation |  | erived Units |
| :---: | :---: | :---: | :---: | :---: |
| Area (A) |  | $\mathrm{A}=\mathrm{L} 2$ |  |  |
| Volume (V) |  | $\mathrm{V}=\mathrm{L} 3$ |  |  |
| Density ( $\rho$ ) |  | $\rho=\mathrm{m} / \mathrm{V}$ |  | gm $3=\mathrm{kg} \mathrm{m}-3$ |
| Velocity (v) |  | $\mathrm{v}=\mathrm{L} / \mathrm{t}$ |  | $\mathrm{s}=\mathrm{m} \mathrm{s-1}$ |
| Acceleration (a) |  | $\mathrm{a}=\Delta \mathrm{v} / \mathrm{t}$ |  | $\mathrm{s}-1 \mathrm{~s}=\mathrm{mm}$-2 |
| Momentum (p) |  | $\mathrm{p}=\mathrm{mxv}$ |  | $\mathrm{kg})(\mathrm{ms}-1)=\mathrm{kg} \mathrm{m} \mathrm{s}-1$ |
| Derived <br> Quantities | Equation | Derived Unit |  | Derived Units |
|  |  | Special Name | Symbol |  |
| Force (F) | $\mathrm{F}=\square \mathrm{pt}$ | Newton | N | $\mathrm{kg} \mathrm{m} \mathrm{s-1s}=\mathrm{kg} \mathrm{m} \mathrm{s}-2$ |
| Pressure (p) |  |  |  |  |
| Energy (E) | $\mathrm{E}=\mathrm{Fxd}$ | Joule | J | $(\mathrm{kg} \mathrm{m} \mathrm{s}-2)(\mathrm{m})=\mathrm{kg} \mathrm{m} 2 \mathrm{~s}-2$ |
| Power (P) | $\mathrm{P}=\mathrm{Et}$ | Watt | W | $\mathrm{kg} \mathrm{m} 2 \mathrm{~s}-2 \mathrm{~s}=\mathrm{kg} \mathrm{m} 2 \mathrm{~s}-3$ |
| Frequency (f) | $\mathrm{f}=1 \mathrm{t}$ | Hertz | Hz | $1 \mathrm{~s}=\mathrm{s}-1$ |
| Charge (Q) | $\mathrm{Q}=\mathrm{Ix} \mathrm{t}$ | Coulomb | C | A s |
| Potential <br> Difference (V) | $V=E Q$ | Volt | V | $\begin{aligned} & \mathrm{kg} \mathrm{~m} 2 \mathrm{~s}-2 \mathrm{As}=\mathrm{kg} \mathrm{~m} 2 \mathrm{~s}-3 \mathrm{~A}- \\ & 1 \end{aligned}$ |
| Resistance (R) | $\mathrm{R}=\mathrm{VI}$ | Ohm | $\Omega$ | $\begin{aligned} & \mathrm{kg} \mathrm{~m} 2 \mathrm{~s}-3 \mathrm{~A}-1 \mathrm{~A}=\mathrm{kg} \mathrm{~m} 2 \mathrm{~s}- \\ & 3 \mathrm{~A}-2 \end{aligned}$ |

[^0]
## Chapter I: Measurements

### 1.3 Expressing larger and smaller physical quantities

Once the fundamental units are defined, it is easier to express larger and smaller units of the same physical quantity. In the metric (SI) system these are related to the fundamental unit in multiples of 10 or $1 / 10$. Thus 1 km is 1000 m and 1 mm is $1 / 1000$ meter. Table 1.3 lists the standard SI prefixes, their meanings and abbreviations.

| Power of ten | Prefix | Abbreviation |
| :---: | :--- | :--- |
| $10^{-15}$ | femto | f |
| $10^{-12}$ | pico | p |
| $10^{-9}$ | nano | n |
| $10^{-6}$ | Micro | $\mu$ |
| $10^{-3}$ | milli | m |
| $10^{-2}$ | denti | c |
| $10^{-1}$ | hecto | d |
| $10^{1}$ | kilo | da |
| $10^{2}$ | mega | k |
| $10^{3}$ | giga | M |
| $10^{6}$ | tera | G |
| $10^{9}$ | peta | P |
| $10^{12}$ |  |  |
| $10^{15}$ |  |  |

Table 3: standard SI prefixes

## Chapter I: Measurements

## Check your Understanding

To consolidate your information and understanding, try to answer to these MCQs
Answers to the following MCQs are in bold

1. Physical sciences were divided into
a. 4 disciplines
b. 3 disciplines
c. 5 disciplines
d. 6 disciplines
2. When a standard is set for a quantity, then standard quantity is called a
a. Amount
b. Rate
c. Prefix
d. Unit
3. A worldwide system of measurements in which the units of base quantities were introduced is called:
a. Prefixes
b. international system of units
c. sexasigmal system
d. none of above
4. Unit which is not derived is
a. Newton
b. Kilogram
c. Watt
d. Pascal
5. Amount of a substance in terms of numbers is measured in
a. Gram
b. kilogram
c. Mole
d. Newton
6. Units used to measure derived quantities are known as
a. square units
b. derived units
c. base units
d. none of above
7. Thermal energy from a hot body flows to a cold body in form of
a. sound
b. signals
c. heat
d. waves
8. All physical quantities are
a. not measurable
b. measurable
c. related to each other
d. not related to each other
9. Base quantity among following is
a. electric charge
b. amount of substance
c. area
d. volume
10. Volume, area, speed, electric charge, force and work are examples of
a. quartile quantities
b. base quantities
c. derived quantities
d. prefixes
11. Derived quantities can be expressed in form of
a. base quantities
b. physical quantities
c. non measurable quantities
d. both B and C

## Chapter I: Measurements

## Glossary

Accuracy: the degree to which a measured value agrees with correct value for that measurement

Approximation: an estimated value based on prior experience and reasoning
Classical Physics: physics that was developed from the Renaissance to the end of the 19th century

Conversion factor: a ratio expressing how many of one unit are equal to another unit
Derived units: units that can be calculated using algebraic combinations of the fundamental units

English units (imperial): system of measurement used in the United States; includes units of measurement such as feet, gallons, and pounds

Fundamental units: units that can only be expressed relative to the procedure used to measure them

Kilogram: the SI unit for mass, abbreviated (kg)
Law: a description, using concise language or a mathematical formula, a generalized pattern in nature that is supported by scientific evidence and repeated experiments

Meter: the SI unit for length, abbreviated (m)
Method of adding percents: the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation

Metric system: a system in which values can be calculated in factors of 10
Model: representation of something that is often too difficult (or impossible) to display directly
Modern Physics: the study of relativity, quantum mechanics, or both
Order of Magnitude: refers to the size of a quantity as it relates to a power of 10
Percent uncertainty: the ratio of the uncertainty of a measurement to the measured value, expressed as a percentage

Physical quantity: a characteristic or property of an object that can be measured or calculated from other measurements

Physics: the science concerned with describing the interactions of energy, matter, space, and time; it is especially interested in what fundamental mechanisms underlie every phenomenon

Precision: the degree to which repeated measurements agree with each other
Quantum mechanics: the study of objects smaller than can be seen with a microscope
Relativity: the study of objects moving at speeds greater than about $1 \%$ of the speed of light, or of objects being affected by a strong gravitational field

## Chapter I: Measurements

SI units: the international system of units that scientists in most countries have agreed to use; includes units such as meters, liters, and grams

Scientific method: a method that typically begins with an observation and question that the scientist will research; next, the scientist typically performs some research about the topic and then devises a hypothesis; then, the scientist will test the hypothesis by performing an experiment; finally, the scientist analyzes the results of the experiment and draws a conclusion

Second: the SI unit for time, abbreviated (s)
Significant figures: express the precision of a measuring tool used to measure a value
Theory: an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers

Uncertainty: a quantitative measure of how much your measured values deviate from a standard or expected value

Units: a standard used for expressing and comparing measurements

## II. One-dimensional Kinematics

## Kinematics

$d=v t \quad d=\frac{1}{2}\left[v_{0}+V_{F}\right] t$
$x_{F}=x_{0}+\bar{v} t$

$$
V_{F}=V_{0}+a t
$$

$V_{F}{ }^{2}=V_{0}^{2}+\mathrm{rad}$


## Learning objectives :

2 One-dimensional Kinematics

### 2.1 Displacement

### 2.1.1 Position

### 2.1.2 Distance

2.2 Vectors and Scalars Coordinate Systems
2.3 Coordinate Systems of one-dimensional motion
2.4 Time, velocity and speed
2.4.1 Time
2.4.2 Velocity
2.4.3 Speed
2.5 Acceleration
2.6 Falling Objects
2.6.1 Gravity

Check your Understanding
Glossary

## Chapter II: One-dimensional Kinematics

## 2 One-dimensional Kinematics

Objects are in motion everywhere we look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins; even in inanimate objects, there is continuous motion in the vibrations of atoms and molecules. Questions about motion are interesting in and of themselves: How long will it take for a space probe to get to Mars? Where will a football land if it is thrown at a certain angle? But an understanding of motion is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

Our formal study of physics begins with kinematics which is defined as the study of motion without considering its causes. The word "kinematics" comes from a Greek term meaning motion and is related to other English words such as "cinema" (movies) and "kinesiology" (the study of human motion). In one-dimensional kinematics, we will study only the motion of a football, for example, without worrying about what forces cause or change its motion. In this chapter, we examine the simplest type of motion-namely, motion along a straight line, or onedimensional motion.

### 2.1 Displacement

Displacement in Physics is described in terms of position and distance. If an object moves relative to a reference frame, then the object's position changes. This change in position is known as displacement. The word "displacement" implies that an object has moved, or has been displaced.

## Displacement :

Displacement is the change in position of an object:

$$
\Delta x=x f-x 0
$$

where $\Delta \mathrm{x}$ is disnlacement. xf is the final nosition. and x 0 is the initial nosition.

### 2.1.1 Position

In order to describe the motion of an object, we must first be able to describe its position-where it is at any particular time. More precisely, we need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a rocket launch would be described in terms of the position of the rocket with respect to the Earth as a whole. In other cases, we use reference frames that are not stationary but are

## Chapter II: One-dimensional Kinematics

in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame.

### 2.1.2 Distance

Although displacement is described in terms of direction, distance is not. Distance is defined to be the magnitude or size of displacement between two positions. Note that the distance between two positions is not the same as the distance traveled between them. Distance traveled is the total length of the path traveled between two positions. Distance has no direction and, thus, no sign.

### 2.2 Vectors and Scalars Coordinate Systems

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude only. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A vector is any quantity with both magnitude and direction.
The direction of a vector in one-dimensional motion is given simply by a plus ( + ) or minus ( - ) sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector's magnitude (e.g., the larger the magnitude, the longer the length of the vector) and points in the same direction as the vector.

Some physical quantities, like distance, either have no direction or none is specified. A scalar is any quantity that has a magnitude, but no direction.
For example, a $20^{\circ} \mathrm{C}$ temperature, the 250 kilocalories ( 250 Calories) of energy in a candy bar, a $90 \mathrm{~km} / \mathrm{h}$ speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars-quantities with no specified direction. Note, however, that a scalar can be negative, such as a $-20^{\circ} \mathrm{C}$ temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.


Figure 1: speed vs. velocity

## Chapter II: One-dimensional Kinematics

### 2.3 Coordinate Systems of one-dimensional motion

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet in (Figure 2.2), it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.


Figure 2: Direction of motion:It is usually convenient to consider motion upward or to the right as positive ( + ) and motion downward or to the left as negative ( - ).

### 2.4 Time, velocity and speed

There is more to motion than distance and displacement. Questions such as, "How long does a foot race take?" and "What was the runner's speed?" cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

### 2.4.1 Time

The most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in some physical quantity. It may be a number on a digital clock, a heartbeat, or the position of the Sun in the sky. In physics, the definition of time is simple - time is change, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

## Chapter II: One-dimensional Kinematics

The amount of time or change is calibrated by comparison with a standard. The SI unit for time is the second, abbreviated s . We might, for example, observe that a certain pendulum makes one full swing every 0.75 s . We could then use the pendulum to measure time by counting its swings or, of course, by connecting the pendulum to a clock mechanism that registers time on a dial. This allows us to not only measure the amount of time, but also to determine a sequence of events.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. Elapsed time $\Delta \mathbf{t}$ is the difference between the ending time and beginning time,

## $\boldsymbol{\Delta t}=\mathbf{t f} \mathbf{- t 0}$

Where $\Delta t$ is the change in time or elapsed time, tf is the time at the end of the motion, and t 0 is the time at the beginning of the motion. (The delta symbol, $\Delta$, means the change in the quantity that follows it.)

### 2.4.2 Velocity

The notion of velocity is probably the same as its scientific definition. We know that if we have a large displacement in a small amount of time we have a large velocity, and that velocity has units of distance divided by time, such as miles per hour or kilometers per hour.

## Average velocity:

Average velocity is displacement (change in position) divided by the time of travel,

$$
v-=\Delta x / \Delta t=x f-x 0 / t f-t 0
$$

Where v - is the average (indicated by the bar over the v ) velocity, $\Delta \mathrm{x}$ is the change in position (or displacement), and xf and x 0 are the final and beginning positions at times tf and t 0 , respectively. If the starting time t 0 is taken to be zero, then the average velocity is simply:

$$
v-=\Delta x / t
$$

### 2.4.3 Speed

In everyday language, most people use the terms "speed" and "velocity" interchangeably. In physics, however, they do not have the same meaning and they are distinct concepts. One major difference is that speed has no direction. Thus, speed is a scalar. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

## Chapter II: One-dimensional Kinematics

Instantaneous speed is the magnitude of instantaneous velocity, whereas average speed is the distance traveled divided by elapsed time.
We have noted that distance traveled can be greater than displacement. So average speed can be greater than average velocity.

### 2.5 Acceleration

In everyday conversation, to accelerate means to speed up. The accelerator in a car can in fact cause it to speed up. The greater the acceleration, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

## Average acceleration:

Average Acceleration is the rate at which velocity changes

$$
\overline{\mathbf{a}}=\Delta v / \Delta t=v f-v 0 / t f-t 0
$$

Where a - is average acceleration, v is velocity, and t is time. (The bar over the a means average acceleration.)

## Remark:

- Acceleration as a Vector: Acceleration is a vector in the same direction as the change in velocity, $\Delta \mathrm{v}$. Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.
- Although acceleration is in the direction of the change in velocity, it is not always in the direction of motion. When an object slows down, its acceleration is opposite to the direction of its motion. This is known as deceleration.


### 2.6 Falling Objects

Falling objects form an interesting class of motion problems. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. By applying the kinematics developed so far to falling objects, we can examine some interesting situations and learn much about gravity in the process.

### 2.6.1 Gravity

The most remarkable and unexpected fact about falling objects is that, if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the same constant acceleration, independent of their mass. This experimentally determined fact is unexpected, because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones.

## Chapter II: One-dimensional Kinematics



In air


In a vacuum


In a vacuum (the hard way)

Figure 3: a hammer and a feather falling in air (friction and resistance) and in a vaccum
In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. Air resistance and friction oppose the motion of an object through the air.
The force of gravity causes objects to fall toward the center of Earth. The acceleration of freefalling objects is therefore called the acceleration due to gravity. The acceleration due to gravity is constant, which means we can apply the kinematics equations to any falling object where air resistance and friction are negligible. This opens a broad class of interesting situations. The acceleration due to gravity is so important that its magnitude is given its own symbol, $\mathbf{g}$. It is constant at any given location on Earth and has the average value $\mathrm{g}=\mathbf{9 . 8 0} \mathbf{~ m} / \mathrm{s} 2$.

## Check Your Understanding

To consolidate your information and understanding, try to answer to these MCQs
Answers to the following MCQs are in bold.

1. Total distance covered in total time taken is termed as
a. instantaneous speed
b. average speed
c. uniform speed
d. variable speed
2. Velocity is the
a. distance covered per unit time
b. displacement covered per unit time
c. time taken per unit distance
d. time taken per unit displacement
3. Distance in specified direction is termed as
a. Directional Distance
b. Uni-directional Distance
c. Displacement
d. Directional Displacement
4. When speed of object changes, velocity
a. remains same
b. also changes
c. decreases
d. increases
5. One of characteristics of air resistance is
a. It does not oppose the motion
b. It decreases with the speed of the object
c. It decreases with the surface area

## Chapter II: One-dimensional Kinematics

## d. It increases with the density of the air

6. Deceleration is also known as
A. retardation
B. acceleration
C. opposite velocity
D. inertia
7. Air resistance is a
a. frictional force
b. gravitational force
c. backward force
d. balanced force
8. If a feather and an iron bar is released from a same height in a room without any air resistance. first one to fall is
a. Iron bar
b. Feather
c. Both at the same time
d. They won't fall
9. Following that is not characteristic of air resistance is
a. It always opposes the motion
b. It decreases with the speed of the object
c. It increases with the surface area
d. It increases with the density of the air
10. Unchanged or constant speed is termed as
a. instantaneous speed
b. average speed
c. uniform speed
d. variable speed
11. Change of distance in a specified direction per unit time is termed as
a. Acceleration
b. Velocity
c. Speed
d. Directional Speed
12. When speed remains constant, velocity
a. may change
b. remain constant
c. must changes
d. slightly increases
13. Changing or inconsistent speed is termed as
a. instantaneous speed
b. average speed
c. uniform speed
d. variable speed
14. Theory that 'all object falling under gravity accelerate at same constant rate' was discovered by
a. Albert Einstein
b. Robert Hooke
c. sir Isaac Newton
d. Galileo Galilei
15. Negative acceleration is termed as
a. ceasing
b. retardation
c. inertia
d. opposite velocity
16. Acceleration due to free-fall or gravity doesn't depend on
a. size
b. Material
c. Shape and size
d. Shape, size and material
17. Speed that you check for a moment on speed-o-meter is termed as
a. instantaneous speed
b. average speed
c. uniform speed
d. variable speed

## Chapter II: One-dimensional Kinematics

## Glossary

Acceleration due to gravity: acceleration of an object as a result of gravity
Acceleration: the rate of change in velocity; the change in velocity over time
Average acceleration: the change in velocity divided by the time over which it changes
Average speed: distance traveled divided by time during which motion occurs
Average velocity: displacement divided by time over which displacement occurs
Deceleration: acceleration in the direction opposite to velocity; acceleration that results in a decrease in velocity

Dependent variable: the variable that is being measured; usually plotted along the y-axis
Displacement: the change in position of an object
Distance traveled: the total length of the path traveled between two positions
Distance: the magnitude of displacement between two positions
Elapsed time: the difference between the ending time and beginning time
Free-fall: the state of movement that results from gravitational force only
Independent variable: the variable that the dependent variable is measured with respect to; usually plotted along the x -axis

Instantaneous acceleration: acceleration at a specific point in time
Instantaneous speed: magnitude of the instantaneous velocity
Instantaneous velocity: velocity at a specific instant, or the average velocity over an infinitesimal time interval

Kinematics: the study of motion without considering its causes
Model: simplified description that contains only those elements necessary to describe the physics of a physical situation

Position: the location of an object at a particular time
Scalar: a quantity that is described by magnitude, but not direction
Slope: the difference in $y$-value (the rise) divided by the difference in $x$-value (the run) of two points on a straight line

Time: change, or the interval over which change occurs
Vector: a quantity that is described by both magnitude and direction
Y-intercept: the y - value when $\mathrm{x}=0$, or when the graph crosses the y -axis

## Chapter III: Two dimension Kinematics

## II. Two-dimensional Kinematics

## Projectile Motion



[^1]
## Chapter III: Two dimension Kinematics

## 3 Two dimension Kinematics

The arc of a basketball, the orbit of a satellite, a bicycle rounding a curve, a swimmer diving into a pool, blood gushing out of a wound, and a puppy chasing its tail are but a few examples of motions along curved paths. In fact, most motions in nature follow curved paths rather than straight lines.

Motion along a curved path on a flat surface or a plane (such as that of a ball on a pool table or a skater on an ice rink) is two-dimensional, and thus described by two-dimensional kinematics. Motion not confined to a plane, such as a car following a winding mountain road, is described by three-dimensional kinematics. Both two- and three-dimensional kinematics are simple extensions of the one-dimensional kinematics developed for straight-line motion in the previous chapter. This simple extension will allow us to apply physics to many more situations, and it will also yield unexpected insights about nature.

- The shortest path between any two points is a straight line. In two dimensions, this path can be represented by a vector with horizontal and vertical components as in figure 3.1.
- The horizontal and vertical components of a vector are independent of one another. Motion in the horizontal direction does not affect motion in the vertical direction, and vice versa. Figure 3.4.


Figure 4: The straight-line path followed by a helicopter between the two points is shorter than the 14 blocks walked by the pedestrian.

## Chapter III: Two dimension Kinematics



Figure 5: independence of motion

### 3.1 Vector Addition and Subtraction: Graphical Methods

- The graphical method of adding vectors A and B involves drawing vectors on a graph and adding them using the head-to-tail method. The resultant vector $\mathbf{R}$ is defined such that $\mathbf{A}+\mathbf{B}=\mathbf{R}$. The magnitude and direction of $\mathbf{R}$ are then determined with a ruler and protractor, respectively.
- The graphical method of subtracting vector $B$ from $A$ involves adding the opposite of vector $\mathbf{B}$, which is defined as $-\mathbf{B}$. In this case, $\mathbf{A}-\mathbf{B}=\mathbf{A}+(-\mathbf{B})=\mathbf{R}$. Then, the head-to-tail method of addition is followed in the usual way to obtain the resultant vector $\mathbf{R}$.
- Addition of vectors is commutative such that $\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$.
- The head-to-tail method of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.
- If a vector A is multiplied by a scalar quantity $\boldsymbol{c}$, the magnitude of the product is given by $c \mathrm{~A}$. If c is positive, the direction of the product points in the same direction as $\mathbf{A}$; if c is negative, the direction of the product points in the opposite direction as $\mathbf{A}$.


### 3.2 Vector Addition and Subtraction: Analytical Methods

Magnitude and direction of a resultant vector.

- The steps to add vectors $\mathbf{A}$ and $\mathbf{B}$ using the analytical method are as follows:

Step 1: Determine the coordinate system for the vectors. Then, determine the horizontal and vertical components of each vector using the equations

## Chapter III: Two dimension Kinematics

$$
\begin{array}{lll}
\mathbf{A}_{\mathbf{x}}=\mathrm{A} \cos \theta \\
\mathbf{B}_{\mathbf{x}}=\mathrm{B} \cos \theta & \text { and } & \mathbf{A}_{\mathbf{y}}=\mathrm{A} \sin \theta \\
& \mathbf{B}_{\mathbf{y}}=\mathrm{B} \sin \theta
\end{array}
$$

Step 2: Add the horizontal and vertical components of each vector to determine the components $\mathrm{R}_{\mathrm{x}}$ and $\mathrm{R}_{\mathrm{y}}$ of the resultant vector, $\mathbf{R}$ :
$\mathbf{R}_{\mathrm{x}}=\mathrm{A}_{\mathrm{x}}+\mathrm{B}_{\mathrm{x}}$
and
$\mathbf{R}_{\mathbf{y}}=\mathrm{A}_{\mathrm{y}}+\mathrm{B}_{\mathrm{y}}$.
Step 3: Use the Pythagorean theorem to determine the magnitude, $\mathbf{R}$, of the resultant vector $\mathbf{R}$ :
$\mathbf{R}=\mathrm{R}_{\mathrm{x}} 2+\mathrm{R}_{\mathrm{y}} 2$.
Step 4: Use a trigonometric identity to determine the direction, $\boldsymbol{\theta}$, of R :
$\boldsymbol{\theta}=\tan ^{-1}\left(\mathrm{R}_{\mathrm{y}} / \mathrm{R}_{\mathrm{x}}\right)$.

### 3.3 Projectile Motion

Projectile motion is the motion of an object thrown or projected into the air, subject to only the acceleration of gravity. The object is called a projectile, and its path is called its trajectory. The motion of falling objects is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In this section, we consider two-dimensional projectile motion, such as that of a football for which air resistance is negligible (figure 3.3) The most important fact to remember here is that motions along perpendicular axes are independent and thus can be analyzed separately. The key to analyzing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. (This choice of axes is the most sensible, because acceleration due to gravity is vertical-thus, there will be no acceleration along the horizontal axis when air resistance is negligible.) As is customary, we call the horizontal axis the x -axis and the vertical axis the y axis. The magnitudes of these vectors are $\mathbf{s}, \mathbf{x}$, and $\mathbf{y}$.


Figure 6: The total displacement s of a soccer ball at a point along its path. The vector $s$ has components $x$ and $y$ along the horizontal and vertical axes. Its magnitude is $s$, and it makes an angle $\vartheta$ with the horizontal

## Chapter III: Two dimension Kinematics

- To solve projectile motion problems, perform the following steps:

1. Determine a coordinate system. Then, resolve the position and/or velocity of the object in the horizontal and vertical components. The components of position $s$ are given by the quantities $\mathbf{x}$ and $\mathbf{y}$, and the components of the velocity v are given by $\mathbf{v}_{\mathbf{x}}=\mathbf{v} \cos \boldsymbol{\theta}$ and $\mathbf{v}_{\mathbf{y}}=\mathbf{v} \boldsymbol{\operatorname { s i n }} \boldsymbol{\theta}$, where v is the magnitude of the velocity and $\theta$ is its direction.
2. Analyze the motion of the projectile in the horizontal direction using the following equations:

Horizontal motion ( $\mathrm{a}_{\mathrm{x}}=0$ )
$\mathrm{x}=\mathrm{x}_{0}+\mathrm{v}_{\mathrm{x}} \mathrm{t}$
$\mathrm{v}_{\mathrm{x}}=\mathrm{v}_{0 \mathrm{x}}=\mathrm{v}_{\mathrm{x}}=$ velocity is a constant.
3. Analyze the motion of the projectile in the vertical direction using the following equations:

Vertical motion (Assuming positive direction is up; $\mathrm{a}_{\mathrm{y}}=-\mathrm{g}=-9.80 \mathrm{~m} / \mathrm{s} 2$ )
$y=y_{0}+1 / 2\left(v_{0 y}+v_{y}\right) t$
$v_{y}=v_{0 y}-g_{t}$
$y=y_{0}+v_{0 y} t-1 / 2 g t^{2}$
$v^{2} y^{2}=v^{2}{ }_{0 y}-2 \mathrm{~g}\left(\mathrm{y}-\mathrm{y}_{0}\right)$.
4. Recombine the horizontal and vertical components of location and/or velocity using the following equations:
$\mathrm{s}=\sqrt{ } \mathrm{x}^{2}+\mathrm{y}^{2}$
$\theta=\tan ^{-1}(\mathrm{y} / \mathrm{x})$
$\mathrm{v}=\sqrt{ } \mathrm{v}^{2}{ }_{\mathrm{x}}+\mathrm{v}^{2} \mathrm{y}$
$\theta_{\mathrm{v}}=\tan ^{-1}\left(\mathrm{v}_{\mathrm{y}} / \mathrm{v}_{\mathrm{x}}\right)$.

- The maximum height $h$ of a projectile launched with initial vertical velocity $v_{0 y}$ is given by
$\mathrm{h}=\mathrm{v}^{2}{ }_{0 y} / 2 \mathrm{~g}$
- The maximum horizontal distance traveled by a projectile is called the range. The range R of a projectile on level ground launched at an angle
$\theta_{0}$ above the horizontal with initial speed $v 0$ is given by
$R=v^{2}{ }_{0} \sin 2 \theta_{0} / g$


## Chapter III: Two dimension Kinematics

## Check Your Understanding

To consolidate your information and understanding, try to answer to these MCQs
Answers to the following MCQs are in bold.

1. Point of intersection of two coordinate axes is called
A. mid
B. origin
C. circumference
D. radius
2. A vector having 'magnitude only' is called
A. scalar
B. resultant
C. unit vector
D. temperature
3. If value of moment arm is zero, then torque produced will be
A. 1
B. 0
C. doubled
D. decreased

## 4. Product of velocity and mass is

 calledA. momentum
B. work
C. acceleration
D. speed
5. Vector product of two vectors is also known as
A. scalar product
B. dot product
C. point product
D. cross product
6. Sum of all torques acting on a body is zero, this condition represents equilibrium's
A. first condition
B. second condition
C. third condition
D. fourth condition
7. A body at rest or moving with uniform velocity will have acceleration
A. 1
B. 0
C. $\min$
D. $\max$
8. A vector whose magnitude is zero is called a
A. scalar
B. resultant
C. unit vector
D. null vector
9. If two vectors have same magnitude and are parallel to each other, then they are said to be
A. same
B. different
C. negative
D. equal
10. If $2^{\text {nd }}$ condition of equilibrium is satisfied, body will be in
A. translational equilibrium
B. rotational equilibrium
C. static equilibrium
D. dynamic equilibrium
11. Physical quantities having magnitude only are called
A. vector quantities
B. scalar quantities
C. mental quantities

## Chapter III: Two dimension Kinematics

D. both a and b
12. According to system international, unit of torque is
A. newton meter
B. newton per meter
C. newton per meter square
D. per newton meter
13. If first condition of equilibrium is satisfied, then body will be in
A. translational equilibrium
B. rotational equilibrium
C. static equilibrium
D. dynamic equilibrium
14. To satisfy first condition of equilibrium, if rightward forces are positive, leftward forces must be
A. positive
B. negative
C. doubled
D. halved
15. Vector whose magnitude is zero has a
A. positive direction
B. arbitrary direction
C. negative direction
D. both a and b
16. Physical quantity that has magnitude and direction as well is known as
A. mass
B. time
C. velocity
D. temperature
17. Scalar product of two vectors is also known as
A. vector product
B. dot product
C. point product
D. both a and b
18. If a body is in rest or in uniform velocity, it is said to be in
A. rest
B. uniform motion
C. equilibrium
D. constant force
19. Two vectors are considered to be equal if they have same magnitude and
A. different direction
B. positive direction
C. same direction
D. negative direction
20. Distance from a point around which body rotates and point at end is called
A. length of the object
B. moment arm
C. momentum arm
D. distance of object

Chapter IV: Dynamics: Newton's Laws of Motion

## IV. DYNAMICS: FORCE AND NEWTON'S LAWS OF MOTION

## Examples of Newton's Laws



First Law
An object wants to remain in its
current state


Second Law
The acceleration of
an object generates a force in the same direction


Third Law Applying force to an object results in an equal and opposite force


4 Introduction to Dynamics: Newton's Laws of Motion
4.1 Development of Force Concept
4.2 Newton's First Law of Motion: Inertia
4.3 Newton's Second Law of Motion: Concept of a System
4.4 Newton's Third Law of Motion: Symmetry in Forces
4.5 Further Applications of Newton's Laws of Motion
4.6 Extended Topic: The Four Basic Forces

Check your Understanding
Glossary

## Chapter IV: Dynamics: Newton's Laws of Motion

## 4 Introduction to Dynamics: Newton's Laws of Motion

The study of motion is kinematics, but kinematics only describes the way objects move-their velocity and their acceleration. Dynamics considers the forces that affect the motion of moving objects and systems. Newton's laws of motion are the foundation of dynamics. These laws provide an example of the breadth and simplicity of principles under which nature functions. They are also universal laws in that they apply to similar situations on Earth as well as in space.

### 4.1 Development of Force Concept

Dynamics is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. Our intuitive definition of force-that is, a push or a pull-is a good place to start. We know that a push or pull has both magnitude and direction (therefore, it is a vector quantity) and can vary considerably in each regard.

In contrast, Earth exerts only a tiny downward pull on a flea. Our everyday experiences also give us a good idea of how multiple forces add. If two people push in different directions on a third person, we might expect the total force to be in the direction shown. Since force is a vector, it adds just like other vectors. Forces, like other vectors, are represented by arrows and can be added using the familiar head-to-tail method or by trigonometric methods.

A quantitative definition of force can be based on some standard force, just as distance is measured in units relative to a standard distance.

One possibility is to stretch a spring a certain fixed distance, as illustrated in Figure 4.1, and use the force it exerts to pull itself back to its relaxed shape-called a restoring force-as a standard. The magnitude of all other forces can be stated as multiples of this standard unit of force.


Figure 7: Force exerted by a stretched spring

## Chapter IV: Dynamics: Newton's Laws of Motion

### 4.2 Newton's First Law of Motion: Inertia

Experience suggests that an object at rest will remain at rest if left alone, and that an object in motion tends to slow down and stop unless some effort is made to keep it moving. What Newton's first law of motion states, however, is the following:

Newton's First Law of Motion:
A body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force

Newton's first law is completely general and can be applied to anything from an object sliding on a table to a satellite in orbit to blood pumped from the heart. Experiments have thoroughly verified that any change in velocity (speed or direction) must be caused by an external force. The idea of generally applicable or universal laws is important not only here-it is a basic feature of all laws of physics. Identifying these laws is like recognizing patterns in nature from which further patterns can be discovered.

## Summary:

- Newton's first law of motion states that a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force. This is also known as the law of inertia.
- Inertia is the tendency of an object to remain at rest or remain in motion. Inertia is related to an object's mass.
- Mass is the quantity of matter in a substance.


### 4.3 Newton's Second Law of Motion: Concept of a System

Newton's second law of motion is closely related to Newton's first law of motion. It mathematically states the cause and effect relationship between force and changes in motion. Newton's second law of motion is more quantitative and is used extensively to calculate what happens in situations involving a force.

- Change in motion is equivalent to a change in velocity. A change in velocity means, by definition, that there is an acceleration. Newton's first law says that a net external force causes a change in motion; thus, we see that a net external force causes acceleration.
- An external force acts from outside the system of interest, figure 4.2


## Chapter IV: Dynamics: Newton's Laws of Motion



Figure 8: Different forces exerted on the same mass produce different accelerations

## Newton's Second Law of Motion:

The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.

In equation form, Newton's second law of motion is

$$
\mathrm{a}=\mathbf{F}_{\mathrm{net}} / \mathrm{m} .
$$

This is often written in the more familiar form
$\mathrm{F}_{\text {net }}=\mathrm{ma}$.
When only the magnitude of force and acceleration are considered, this equation is simply
$\mathbf{F}_{\text {net }}=\mathbf{m a}$.

- The weight $w$ of an object is defined as the force of gravity acting on an object of mass $\mathbf{m}$. The object experiences an acceleration due to gravity $\mathbf{g}$ :
$\mathbf{w}=\mathrm{mg}$.
- If the only force acting on an object is due to gravity, the object is in free fall.
- Friction is a force that opposes the motion past each other of objects that are touching
- When the net external force on an object is its weight, we say that it is in free-fall.


## Chapter IV: Dynamics: Newton's Laws of Motion

### 4.4 Newton's Third Law of Motion: Symmetry in Forces

This law represents a certain symmetry in nature: Forces always occur in pairs, and one body cannot exert a force on another without experiencing a force itself. We sometimes refer to this law loosely as "action-reaction," where the force exerted is the action and the force experienced as a consequence is the reaction. Newton's third law has practical uses in analyzing the origin of forces and understanding which forces are external to a system.

## Newton's Third Law of Motion:

Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts.

### 4.5 Further Applications of Newton's Laws of Motion

- Newton's laws of motion can be applied in numerous situations to solve problems of motion.
- Some problems will contain multiple force vectors acting in different directions on an object. For this, we need to draw diagrams, resolve all force vectors into horizontal and vertical components, and draw a free-body diagram. Always analyze the direction in which an object accelerates so that you can determine whether $\mathrm{F}_{\text {net }}=\mathrm{ma}$ or $\mathrm{F}_{\text {net }}=0$
- The normal force on an object is not always equal in magnitude to the weight of the object. If an object is accelerating, the normal force will be less than or greater than the weight of the object. In addition, if the object is on an inclined plane, the normal force will always be less than the full weight of the object.
- Some problems will contain various physical quantities, such as forces, acceleration, velocity, or position. You can apply concepts from kinematics and dynamics in order to solve these problems of motion.


### 4.6 Extended Topic: The Four Basic Forces

- The various types of forces that are categorized for use in many applications are all manifestations of the four basic forces in nature.
- The properties of these forces are summarized in Table 4.1.


## Chapter IV: Dynamics: Newton's Laws of Motion

| Force | Approximate <br> Relative <br> Strengths | Range | Attraction/Repulsion | Carrier <br> Particle |
| :--- | :--- | :--- | :--- | :--- |
| Gravitational | $10^{-38}$ | $\infty$ | attractive only | Graviton |
| Electromagnetic | $10^{-2}$ | $\infty$ | Attractive <br> and repulsive | Photon |
| Weak Nuclear | $10^{-13}$ | $<10^{-18} \mathrm{~m}$ | Attractive <br> and repulsive | $\mathrm{W}^{+}, \mathrm{W} \mathrm{Z}^{0}$, <br> Strong Nuclear |

Table 4: Properties of the Four Basic Forces

- Everything we experience directly without sensitive instruments is due to either electromagnetic forces or gravitational forces. The nuclear forces are responsible for the submicroscopic structure of matter, but they are not directly sensed because of their short ranges. Attempts are being made to show all four forces are different manifestations of a single unified force.
- A force field surrounds an object creating a force and is the carrier of that force.


## Check Your Understanding

To consolidate your information and understanding, try to answer to these MCQs
Answers to the following MCQs are in bold.

1. Total distance covered in total time taken is termed as
e. instantaneous speed
f. average speed
g. uniform speed
h. variable speed
2. Velocity is the
e. distance covered per unit time
f. displacement covered per unit time
g. time taken per unit distance
h. time taken per unit displacement
3. Distance in specified direction is termed as
e. Directional Distance
f. Uni-directional Distance
g. Displacement
h. Directional Displacement
4. When speed of object changes, velocity
e. remains same
f. also changes
g. decreases
h. increases
5. One of characteristics of air resistance is
e. It does not oppose the motion
f. It decreases with the speed of the object
g. It decreases with the surface area

## Chapter IV: Dynamics: Newton's Laws of Motion

## h. It increases with the density of the air

6. Deceleration is also known as
E. retardation
F. acceleration
G. opposite velocity
H. inertia
7. Air resistance is a
e. frictional force
f. gravitational force
g. backward force
h. balanced force
8. If a feather and an iron bar is released from a same height in a room without any air resistance. first one to fall is
e. Iron bar
f. Feather
g. Both at the same time
h. They won't fall
9. Following that is not characteristic of air resistance is
e. It always opposes the motion
f. It decreases with the speed of the object
g. It increases with the surface area
h. It increases with the density of the air
10. Unchanged or constant speed is termed as
e. instantaneous speed
f. average speed
g. uniform speed
h. variable speed
11. Change of distance in a specified direction per unit time is termed as
e. Acceleration
f. Velocity
g. Speed
h. Directional Speed
12. When speed remains constant, velocity
e. may change
f. remain constant
g. must changes
h. slightly increases
13. Changing or inconsistent speed is termed as
e. instantaneous speed
f. average speed
g. uniform speed
h. variable speed
14. Theory that 'all object falling under gravity accelerate at same constant rate' was discovered by
e. Albert Einstein
f. Robert Hooke
g. sir Isaac Newton
h. Galileo Galilei
15. Negative acceleration is termed as
e. ceasing
f. retardation
g. inertia
h. opposite velocity
16. Acceleration due to free-fall or gravity doesn't depend on
e. size
f. Material
g. Shape and size
h. Shape, size and material
17. Speed that you check for a moment on speed-o-meter is termed as
e. instantaneous speed
f. average speed
g. uniform speed
h. variable speed

## Chapter IV: Dynamics: Newton's Laws of Motion

## Glossary

Acceleration: The rate at which an object's velocity changes over a period of time
Carrier particle: A fundamental particle of nature that is surrounded by a characteristic force field; photons are carrier particles of the electromagnetic force.

Dynamics: The study of how forces affect the motion of objects and systems
External force: A force acting on an object or system that originates outside of the object or system

Force field: A region in which a test particle will experience a force
Force: A push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force

Free-body diagram: A sketch showing all of the external forces acting on an object or system; the system is represented by a dot, and the forces are represented by vectors extending outward from the dot

Free-fall: A situation in which the only force acting on an object is the force due to gravity
Friction: A force past each other of objects that are touching; examples include rough surfaces and air resistance

Inertia: The tendency of an object to remain at rest or remain in motion
Inertial frame of reference: A coordinate system that is not accelerating; all forces acting in an inertial frame of reference are real forces, as opposed to fictitious forces that are observed due to an accelerating frame of reference

Law of inertia: Newton's first law of motion
Mass: The quantity of matter in a substance; measured in kilograms
Newton's first law of motion: A body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force; also known as the law of inertia

Newton's second law of motion: The net external force Fnet on an object with mass $m$ is proportional to and in the same direction as the acceleration of the object, a , and inversely proportional to the mass; defined mathematically as $\mathbf{a}=$ Fnet $/ \mathbf{m}$

Newton's third law of motion: whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts

Net external force: The vector sum of all external forces acting on an object or system; causes a mass to accelerate

Normal force: The force that a surface applies to an object to support the weight of the object; acts perpendicular to the surface on which the object rests

## Chapter IV: Dynamics: Newton's Laws of Motion

System: defined by the boundaries of an object or collection of objects being observed; all forces originating from outside of the system are considered external forces

Tension: The pulling force that acts along a medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force on the object due to the rope is called a tension force

Thrust: A reaction force that pushes a body forward in response to a backward force; rockets, airplanes, and cars are pushed forward by a thrust reaction force

Weight: The force $\mathbf{w}$ due to gravity acting on an object of mass $\boldsymbol{m}$; defined mathematically as: $\mathrm{w}=\mathrm{mg}$, where $\mathbf{g}$ is the magnitude and direction of the acceleration due to gravity.

Chapter V: Further Applications of Newton's Laws

## V. FURTHER APPLICATIONS OF NEWTON'S LAWS: FRICTION, DRAG, AND ELASTICITY

## Elasticity \& Hooke's Law



5 Further Applications of Newton's Laws
5.1 Friction
5.2 Drag forces
5.3 Elasticity: Stress and Strain
5.4 Changes in Length-Tension and Compression: Elastic Modulus
5.5 Sideways Stress: Shear Modulus

Check your understanding
Glossary

## Chapter V: Further Applications of Newton's Laws

## 5 Further Applications of Newton's Laws

It is difficult to categorize forces into various types (aside from the four basic forces discussed in previous chapter). Knowing that a net force affects the motion, position, and shape of an object, it is useful at this point to look at some particularly interesting and common forces that will provide further applications of Newton's laws of motion. We have in mind the forces of friction, air or liquid drag, and deformation.

### 5.1 Friction

Friction is a force that is around us all the time that opposes relative motion between systems in contact but also allows us to move. While a common force, the behavior of friction is actually very complicated and is still not completely understood. We have to rely heavily on observations for whatever understandings we can gain. However, we can still deal with its more elementary general characteristics and understand the circumstances in which it behaves. One of the simpler characteristics of friction is that it is parallel to the contact surface between systems and always in a direction that opposes motion or attempted motion of the systems relative to each other. If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction. But when objects are stationary, static friction can act between them; the static friction is usually greater than the kinetic friction between the objects. Figure 5.1 shows an example of frictional forces.


Figure 9: Frictional forces, such as $f$, always oppose motion or attempted motion between objects in contact

- The magnitude of static friction $f_{\mathrm{s}}$ is


## $f s \leq \mu_{s} \mathbf{N}$

Where $\boldsymbol{\mu}_{\mathrm{s}}$ is the coefficient of static friction and $\mathbf{N}$ is the magnitude of the normal force.

- The kinetic friction force $f_{\mathrm{k}}$ between systems moving relative to one another is given by $f_{\mathrm{k}}=\mu_{\mathrm{k}} \mathbf{N}$

Where $\mu_{\mathrm{k}}$ is the coefficient of kinetic friction.

## Chapter V: Further Applications of Newton's Laws

### 5.2 Drag forces

Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either a gas or a liquid). You feel the drag force when you move your hand through water. You might also feel it if you move your hand during a strong wind. The faster you move your hand, the harder it is to move. You feel a smaller drag force when you tilt your hand so only the side goes through the air-you have decreased the area of your hand that faces the direction of motion. Like friction, the drag force always opposes the motion of an object. Unlike simple friction, the drag force is proportional to some function of the velocity of the object in that fluid. This functionality is complicated and depends upon the shape of the object, its size, its velocity, and the fluid it is in. For most large objects such as bicyclists, cars, and baseballs not moving too slowly, the magnitude of the drag force $\mathbf{F}_{\mathbf{D}}$ is found to be proportional to the square of the speed of the object. We can write this relationship mathematically as $\mathbf{F}_{\mathbf{D}} \propto \mathbf{v}^{\mathbf{2}}$. When taking into account other factors, this relationship becomes:
$F_{D}=1 / 2 C_{\rho} \mathbf{A v}^{2}$,
Where $\mathbf{C}$ is the drag coefficient, $\mathbf{A}$ is the area of the object facing the fluid, and $\boldsymbol{\rho}$ is the density of the fluid.

### 5.3 Elasticity: Stress and Strain

If a bulldozer pushes a car into a wall, the car will not move but it will noticeably change shape. A change in shape due to the application of a force is a deformation. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed.

- First, the object returns to its original shape when the force is removed-that is, the deformation is elastic for small deformations.
- Second, the size of the deformation is proportional to the force-that is, for small deformations, Hooke's law is obeyed.


## Hook's Law

$$
\mathbf{F}=\mathbf{k} \Delta \mathbf{L}
$$

Where $\mathbf{\Delta L}$ is the amount of deformation (the change in length, for example) produced by the force $\mathbf{F}$, and $\mathbf{k}$ is a proportionality constant that depends on the shape and composition of the object and the direction of the force.

$$
\Delta \mathrm{L}=\mathrm{F} / \mathrm{k}
$$

## Chapter V: Further Applications of Newton's Laws

The proportionality constant $\mathbf{k}$ depends upon a number of factors for the material. For example, a guitar string made of nylon stretches when it is tightened, and the elongation $\boldsymbol{\Delta L}$ is proportional to the force applied (at least for small deformations). Thicker nylon strings and ones made of steel stretch less for the same applied force, implying they have a larger $\mathbf{k}$ (see Figure 5.2). Finally, all three strings return to their normal lengths when the force is removed, provided the deformation is small


Figure 10: The same force, in this case a weight ( $w$ ), applied to three different guitar strings of identical length produces the three different deformations shown as shaded segments. The string on the left is thin nylon, the one in the middle is thicker nylon,

### 5.4 Changes in Length-Tension and Compression: Elastic Modulus

A change in length $\Delta \mathbf{L}$ is produced when a force is applied to a wire or rod parallel to its length $\mathbf{L}_{\mathbf{0}}$, either stretching it (a tension) or compressing it. (See Figure 5.3)

Experiments have shown that the change in length ( $\mathbf{\Delta} \mathbf{L}$ ) depends on only a few variables. As already noted, $\Delta \mathrm{L}$ is proportional to the force F and depends on the substance from which the object is made. Additionally, the change in length is proportional to the original length $\mathbf{L}_{0}$ and inversely proportional to the cross-sectional area of the wire or rod. For example, a long guitar string will stretch more than a short one, and a thick string will stretch less than a thin one. We can combine all these factors into one equation for $\Delta \mathbf{L}$ :

$$
\Delta \mathrm{L}=1 \mathrm{~F} / \mathrm{YA} \mathrm{~L}_{0}
$$

## Chapter V: Further Applications of Newton's Laws

Where $\mathbf{\Delta L}$ is the change in length, $\mathbf{F}$ the applied force, $\mathbf{Y}$ is a factor, called the elastic modulus or Young's modulus, that depends on the substance, $\mathbf{A}$ is the cross-sectional area, and $\mathbf{L}_{\mathbf{0}}$ is the original length

(a)

(b)

Figure 11: (a) Tension. The rod is stretched a length $\Delta L$ when a force is applied parallel to its length. (b) Compression. The same rod is compressed by forces with the same magnitude in the opposite direction. For very small deformations and uniform materials, $\Delta L$

## Stress

The ratio of force to area, $\mathbf{F} / \mathbf{A}$, is defined as stress measured in $\mathbf{N} / \mathbf{m}^{\mathbf{2}}$

## Strain

The ratio of the change in length to length, $\mathbf{\Delta L} / \mathbf{L} 0$, is defined as strain (a unitless quantity). In other words,

$$
\text { stress }=\mathbf{Y} \times \text { strain }
$$

Young Modulus
Young Modulus=stress /strain

$$
=\left(\mathbf{F L}_{0}\right) / \mathbf{A}\left(\mathbf{L}_{\mathbf{n}}-\mathbf{L}_{0}\right)
$$

## Chapter V: Further Applications of Newton's Laws

### 5.5 Sideways Stress: Shear Modulus

The shear modulus is defined as the ratio of shear stress to shear strain. It is also known as the modulus of rigidity. Figure 5.4 illustrates what is meant by a sideways stress or a shearing force. Here the deformation is called $\Delta \mathrm{x}$ and it is perpendicular to L 0 ,rather than parallel as with tension and compression. Shear deformation behaves similarly to tension and compression and can be described with similar equations. The expression for shear deformation is

$$
\Delta x=1 F / S A L_{0}
$$

Where $\mathbf{S}$ is the shear modulus and $\mathbf{F}$ is the force applied perpendicular to $\mathbf{L}_{0}$ and parallel to the cross-sectional area $\mathbf{A}$


Figure 12: Shearing forces, are applied perpendicular to the length $L O$ and parallel to the area $A$, producing a deformation $\Delta x$. Vertical forces are not shown, but it should be kept in mind that in addition to the two shearing forces, $F$, there must be supporting

Shear deformation

$$
\Delta x=1 F / S A L 0
$$

Where $\mathbf{S}$ is the shear modulus and $\mathbf{F}$ is the force applied perpendicular to $\mathbf{L o}$ and parallel to the cross-sectional area $\mathbf{A}$

### 5.6 Changes in Volume: Bulk Modulus

An object will be compressed in all directions if inward forces are applied evenly on all its surfaces as in Figure 5.5. It is relatively easy to compress gases and extremely difficult to compress liquids and solids. For example, air in a wine bottle is compressed when it is corked. But if you try corking a brim-full bottle, you cannot compress the wine-some must be removed if the cork is to be inserted. The reason for these different compressibilities is that atoms and molecules are separated by large empty spaces in gases but packed close together in liquids and solids. To compress a gas, we must force its atoms and molecules closer together. To compress liquids and solids, we must actually compress their atoms and molecules, and very strong electromagnetic forces in them oppose this compression.

## Chapter V: Further Applications of Newton's Laws



Figure 13: An inward force on all surfaces compresses this cube. Its change in volume is proportional to the force per unit area and its original volume, and is related to the compressibility of the substance.

We can describe the compression or volume deformation of an object with an equation. First, we note that a force "applied evenly" is defined to have the same stress, or ratio of force to area F/A on all surfaces. The deformation produced is a change in volume $\Delta \mathbf{v}$, which is found to behave very similarly to the shear, tension, and compression previously discussed. (This is not surprising, since a compression of the entire object is equivalent to compressing each of its three dimensions.) The relationship of the change in volume to other physical quantities is given by:

$$
\Delta V=\mathbf{1 F} / \mathbf{B} \mathbf{A} V_{0},
$$

Where $\mathbf{B}$ is the bulk modulus, $\mathbf{V}_{\mathbf{0}}$ is the original volume, and $\mathbf{F} / \mathbf{A}$ is the force per unit area applied uniformly inward on all surfaces

## Bulk Modulus

$$
B=\Delta \mathbf{P} /\left(\Delta \mathrm{v} / \mathbf{V}_{\mathbf{0}}\right)
$$

Where $\Delta \mathbf{P}$ is the change of the pressure or force applied per unit area on the material, $\Delta \mathrm{v}$ is the change of the volume of the material due to the compression and $\mathbf{V}_{\mathbf{0}}$ is the initial volume of the material in the units of in the English system and $\mathrm{N} / \mathrm{m}^{2}$ in the metric system

## Chapter V: Further Applications of Newton's Laws

## Check Your Understanding

Task1: Answer the following questions

1. Any object can be deformed by applying a force to the object. An object is elastic if
$\qquad$ after the force is removed. However, if the object has been deformed beyond its $\qquad$ , it will remain in a deformed state.
2. The stress on an object is defined as the ratio of the $\qquad$ to the $\qquad$ .
3. The strain of an object is defined as the ratio of the $\qquad$ to the $\qquad$ .
4. The generalized statement of Hooke's law states that $\qquad$ is proportional to $\qquad$ of the system.
5. Young's modulus is a constant with units of $\qquad$ , which characterizes the response of a solid to $\qquad$ and which has a magnitude equal to $\qquad$ .
6. The bulk modulus of a material is a constant with units of $\qquad$ , which characterizes the response of a material to $\qquad$ .
7. The shear modulus, also called the $\qquad$ , is a constant with units of $\qquad$ , which characterize the response of a material to $\qquad$ and which has a magnitude equal to
$\qquad$ —.

## Keys:

1. It returns to its original state, elastic limit
2. Applied force, cross-sectional area
3. Change in a spatial variable, original value of that variable
4. Strain, stress
5. $\mathrm{N} / \mathrm{m}^{2}$, forces applied to change its length, FL/A $\Delta \mathrm{L}$
6. $\mathrm{N} / \mathrm{m}^{2}$, forces applied to change its volume without changing its shape, FV/A $\Delta L$
7. Modulus of rigidity, $\mathrm{N} / \mathrm{m} 2$ force applied to change its shape without changing its volume, (F/A) $\varphi$

## Task2:

1. The modulus of elasticity is dimensionally equivalent to
A. strain
B. stress
C. surface tension
D. Poisson's ratio
2. If by applying a force, the shape of a body is changed, then the corresponding stress is known as
A. Tensile stress
B. Bulk stress
C. Shearing stress
D. Compressive stress

## Chapter V: Further Applications of Newton's Laws

3. According to Hooke's law of elasticity, within elastic limits, if the stress is increased, the ratio of stress to strain
A. increases
B. decreases
C. becomes zero
D. remains constant
4. Which one of he following does not affect the elasticity of a substance?
A. Hammering
B. Adding impurity to the substance
C. Changing the dimensions
D. Change in the temperature
5. The bulk modulus of a fluid is inversely proportional to the
A. Change in pressure
B. Volume of the fluid
C. Density of the fluid
D. Change in its volume
6. Shearing strain is given by:
A. Deforming force
B. Shape of shear
C. Angle of shear
D. Change in volume of the body
7. When impurities are added to an elastic substance, its elasticity
A. increases
B. decreases
C. becomes zero
D. maybe increases or decreases
8. If a material is heated and annealed, then its elasticity is
A. increased
B. decreased
C. not changed
D. becoming zero
9. The Young modulus for a plastic body is
A. one
B. zero
C. infinity
D. less than one

## Glossary

Deformation: change in shape due to the application of force
Drag force: $\mathbf{F}_{\mathbf{D}}$, found to be proportional to the square of the speed of the object; mathematically:

$$
\begin{gathered}
\text { FD } \propto v^{2} \\
\text { FD }=\mathbf{1} / \mathbf{2} \boldsymbol{C} \boldsymbol{\rho A} v^{2}
\end{gathered}
$$

Friction: a force that opposes relative motion or attempts at motion between systems in contact

Hooke's law: proportional relationship between the force $\mathbf{F}$ on a material and the deformation $\boldsymbol{\Delta L}$ it causes, $\mathbf{F}=\mathbf{k} \boldsymbol{\Delta L}$

Kinetic friction: a force that opposes the motion of two systems that are in contact and moving relative to one another

Magnitude of kinetic friction: $\mathbf{f}_{\mathbf{k}}=\boldsymbol{\mu}_{\mathbf{k}} \mathbf{N}$, where $\boldsymbol{\mu}_{\mathrm{k}}$ is the coefficient of kinetic friction
Magnitude of static friction: $\mathbf{f}_{s} \leq \boldsymbol{\mu}_{\mathrm{s}} \mathbf{N}$, where $\boldsymbol{\mu}_{\mathrm{s}}$ is the coefficient of static friction and $\mathbf{N}$ is the magnitude of the normal force

## Chapter V: Further Applications of Newton's Laws

Stokes' law: Fs = $\boldsymbol{6} \boldsymbol{\pi} \mathbf{r} \boldsymbol{\eta} \mathbf{v}$, where $\mathbf{r}$ is the radius of the object, $\boldsymbol{\eta}$ is the viscosity of the fluid, and $\mathbf{v}$ is the object's velocity

Shear deformation: deformation perpendicular to the original length of an object
Static friction: a force that opposes the motion of two systems that are in contact and are not moving relative to one another

Strain: ratio of change in length to original length
Stress: ratio of force to area
Tensile strength: measure of deformation for a given tension or compression

Chapter VI: Oscillatory Systems and Waves

## VI. OSCILLATORY MOTION AND WAVES



6 Introduction to oscillatory systems and waves
6.1 Hooke's Law: Stress and Strain Revisited
6.2 Energy in Hooke's Law of Deformation
6.3 Period and Frequency in Oscillations
6.4 Simple Harmonic Motion: A Special Periodic Motion
6.5 The Simple Pendulum
6.6 Uniform Circular Motion and Simple Harmonic Motion
6.7 Damped Harmonic Motion
6.8 Forced Oscillations and Resonance
6.9 Waves
6.9.1 Transverse and Longitudinal Waves

Check your understanding
Glossary

## Chapter VI: Oscillatory Systems and Waves

## 6 Introduction to oscillatory systems and waves

What do an ocean buoy, a child in a swing, the cone inside a speaker, a guitar, atoms in a crystal, the motion of chest cavities, and the beating of hearts all have in common? They all oscillate--that is, they move back and forth between two points. Many systems oscillate, and they have certain characteristics in common. All oscillations involve force and energy. We push a child in a swing to get the motion started. The energy of atoms vibrating in a crystal can be increased with heat. We put energy into a guitar string when you pluck it. Some oscillations create waves. A guitar creates sound waves. We can make water waves in a swimming pool by slapping the water with your hand.

We can no doubt think of other types of waves. Some, such as water waves, are visible. Some, such as sound waves, are not. But every wave is a disturbance that moves from its source and carries energy. Other examples of waves include earthquakes and visible light. Even subatomic particles, such as electrons, can behave like waves.

By studying oscillatory motion and waves, we shall find that a small number of underlying principles describe all of them and that wave phenomena are more common than we have ever imagined. We begin by studying the type of force that underlies the simplest oscillations and waves. We will then expand our exploration of oscillatory motion and waves to include concepts such as simple harmonic motion, uniform circular motion, and damped harmonic motion. Finally, we will explore what happens when two or more waves share the same space, in the phenomena known as superposition and interference.

### 6.1 Hooke's Law: Stress and Strain Revisited

Newton's first law implies that an object oscillating back and forth is experiencing forces. Without force, the object would move in a straight line at a constant speed rather than oscillate. Consider, for example, plucking a plastic ruler to the left as shown in Figure 6.1. The deformation of the ruler creates a force in the opposite direction, known as a restoring force. Once released, the restoring force causes the ruler to move back toward its stable equilibrium position, where the net force on it is zero. However, by the time the ruler gets there, it gains momentum and continues to move to the right, producing the opposite deformation. It is then forced to the left, back through equilibrium, and the process is repeated until dissipative forces dampen the motion. These forces remove mechanical energy from the system, gradually reducing the motion until the ruler comes to rest.

## Chapter VI: Oscillatory Systems and Waves



Figure 14: plastic ruler oscillations
The simplest oscillations occur when the restoring force is directly proportional to displacement. When stress and strain were covered in Newton's Third Law of Motion, the name was given to this relationship between force and displacement was Hooke's law:
$\mathbf{F}=-\mathbf{k x}$.
Here, $\mathbf{F}$ is the restoring force, $\mathbf{x}$ is the displacement from equilibrium or deformation, and $\mathbf{k}$ is a constant related to the difficulty in deforming the system. The minus sign indicates the restoring force is in the direction opposite to the displacement. Equilibrium position


Figure 15: ruler equilibrium position

Explanation: (a) The plastic ruler has been released, and the restoring force is returning the ruler to its equilibrium position. (b) The net force is zero at the equilibrium position, but the ruler has momentum and continues to move to the right. (c) The restoring force is in the opposite direction. It stops the ruler and moves it back toward equilibrium again. (d) Now the ruler has momentum to the left. (e) In the absence of damping (caused by frictional forces), the ruler reaches its original position. From there, the motion will repeat itself.

## Chapter VI: Oscillatory Systems and Waves

The force constant k is related to the rigidity (or stiffness) of a system - the larger the force constant, the greater the restoring force, and the stiffer the system. The units of $\mathbf{k}$ are newtons per meter ( $\mathbf{N} / \mathbf{m}$ ). For example, $\mathbf{k}$ is directly related to Young's modulus when we stretch a string.

### 6.2 Energy in Hooke's Law of Deformation

In order to produce a deformation, work must be done. That is, a force must be exerted through a distance, whether you pluck a guitar string or compress a car spring. If the only result is deformation, and no work goes into thermal, sound, or kinetic energy, then all the work is initially stored in the deformed object as some form of potential energy. The potential energy stored in a spring is $\mathbf{P E} \mathbf{e l}=\mathbf{1} / \mathbf{2} \mathbf{k x}^{\mathbf{2}}$. Here, we generalize the idea to elastic potential energy for a deformation of any system that can be described by Hooke's law. Hence,

$$
\mathrm{PE}_{\mathrm{el}}=1 / 2 \mathbf{k} \mathbf{x}^{2}
$$

Where $\mathbf{P E e l}_{\text {el }}$ is the elastic potential energy stored in any deformed system that obeys Hooke's law and has a displacement $\mathbf{x}$ from equilibrium and a force constant $\mathbf{k}$.

### 6.3 Period and Frequency in Oscillations

When we pluck a guitar string, the resulting sound has a steady tone and lasts a long time. Each successive vibration of the string takes the same time as the previous one. We define periodic motion to be a motion that repeats itself at regular time intervals (figure 6.3), such as exhibited by the guitar string or by an object on a spring moving up and down. The time to complete one oscillation remains constant and is called the period T. Its units are usually seconds, but may be any convenient unit of time. The word period refers to the time for some event whether repetitive or not; but we shall be primarily interested in periodic motion, which is by definition repetitive. A concept closely related to period is the frequency of an event. For example, if you get a paycheck twice a month, the frequency of payment is two per month and the period between checks is half a month. Frequency $f$ is defined to be the number of events per unit time. For periodic motion, frequency is the number of oscillations per unit time. The relationship between frequency and period is

$$
f=1 / T
$$

A cycle is one complete oscillation. Note that a vibration can be a single or multiple event, whereas oscillations are usually repetitive for a significant number of cycles.

## Chapter VI: Oscillatory Systems and Waves



Figure 16: guitar strings vibrating

### 6.4 Simple Harmonic Motion: A Special Periodic Motion

The oscillations of a system in which the net force can be described by Hooke's law are of special importance, because they are very common. They are also the simplest oscillatory systems. Simple Harmonic Motion (SHM) is the name given to oscillatory motion for a system where the net force can be described by Hooke's law, and such a system is called a simple harmonic oscillator. If the net force can be described by Hooke's law and there is no damping (by friction or other non-conservative forces), then a simple harmonic oscillator will oscillate with equal displacement on either side of the equilibrium position, as shown for an object on a spring in Figure 6.4. The maximum displacement from equilibrium is called the amplitude X . The units for amplitude and displacement are the same, but depend on the type of oscillation. For the object on the spring, the units of amplitude and displacement are meters; whereas for sound oscillations, they have units of pressure (and other types of oscillations have yet other units). Because amplitude is the maximum displacement, it is related to the energy in the oscillation.

## Chapter VI: Oscillatory Systems and Waves



Figure 17: an object attached to a spring sliding on a frictionless surface is an uncomplicated simple harmonic oscillator. When displaced from equilibrium, the object performs simple harmonic motion that has an amplitude $\boldsymbol{X}$ and a period $\boldsymbol{T}$. The object's maximum speed occurs as it passes through equilibrium. The stiffer the spring is, the smaller the period $\boldsymbol{T}$. The greater the mass of the object is, the greater the period $\boldsymbol{T}$

What is so significant about simple harmonic motion? One special thing is that the period $\mathbf{T}$ and frequency $f$ of a simple harmonic oscillator are independent of amplitude. The string of a guitar, for example, will oscillate with the same frequency whether plucked gently or hard. Because the period is constant, a simple harmonic oscillator can be used as a clock.

Two important factors do affect the period of a simple harmonic oscillator. The period is related to how stiff the system is. A very stiff object has a large force constant $\mathbf{k}$, which causes the system to have a smaller period. For example, you can adjust a diving board's stiffness-the stiffer it is, the faster it vibrates, and the shorter its period. Period also depends on the mass of the oscillating system. The more massive the system is, the longer the period. For example, a heavy person on a diving board bounces up and down more slowly than a light one.

In fact, the mass $m$ and the force constant $\mathbf{k}$ are the only factors that affect the period and frequency of simple harmonic motion.

## Chapter VI: Oscillatory Systems and Waves

## Period of Simple Harmonic Oscillator

The period of a simple harmonic oscillator is given by:

$$
\mathbf{T}=2 \pi \sqrt{ } \mathrm{~m} / \mathrm{k}
$$

and, because $f=\mathbf{1} / \mathbf{T}$, the frequency of a simple harmonic oscillator is
$f=1 / 2 \boldsymbol{\pi} \sqrt{ } \mathbf{k} / \mathrm{m}$
Note that neither $\mathbf{T}$ nor $\boldsymbol{f}$ has any dependence on amplitude

### 6.5 The Simple Pendulum

Pendulums are in common usage. Some have crucial uses, such as in clocks; some are for fun, such as a child's swing; and some are just there, such as the sinker on a fishing line. For small displacements, a pendulum is a simple harmonic oscillator. A simple pendulum is defined to have an object that has a small mass, also known as the pendulum bob, which is suspended from a light wire or string, such as shown in Figure 6.5.

Exploring the simple pendulum a bit further, we can discover the conditions under which it performs simple harmonic motion, and we can derive an interesting expression for its period.


Figure 18: Simple pendulum
We begin by defining the displacement to be the arc length $\mathbf{s}$. We see from Figure 6.5 that the net force on the bob is tangent to the arc and equals $-\boldsymbol{m} \boldsymbol{g} \boldsymbol{\operatorname { s i n }} \boldsymbol{\theta}$. (The weight $\boldsymbol{m g}$ has components $\boldsymbol{m g} \boldsymbol{\operatorname { c o s }} \boldsymbol{\theta}$ along the string and $\mathrm{mg} \sin \theta$ tangent to the arc.) Tension in the string exactly cancels the component $\mathrm{mg} \cos \theta$ parallel to the string. This leaves a net restoring force back toward the equilibrium position at $\boldsymbol{\theta}=\mathbf{0}$.

Now, if we can show that the restoring force is directly proportional to the displacement, then we have a simple harmonic oscillator. In trying to determine if we have a simple harmonic oscillator, we should note that for small angles (less than about $\left.15^{\circ}\right), \sin \theta \approx \theta(\sin \theta$ and $\theta$

## Chapter VI: Oscillatory Systems and Waves

differ by about $1 \%$ or less at smaller angles). Thus, for angles less than about $15^{\circ}$, the restoring force F is

$$
F \approx-m g \theta .
$$

The displacement $\boldsymbol{s}$ is directly proportional to $\boldsymbol{\theta}$. When $\boldsymbol{\theta}$ is expressed in radians, the arc length in a circle is related to its radius ( L in this instance) by:

$$
\mathbf{s}=\mathbf{L} \boldsymbol{\theta}
$$

so that

$$
\theta=s / \mathbf{L}
$$

For small angles, then, the expression for the restoring force is:

$$
\mathrm{F} \approx-m g / \mathrm{L} s
$$

This expression is of the form:

$$
\boldsymbol{F}=-\mathbf{k x},
$$

Where the force constant is given by $\mathrm{k}=\mathrm{mg} / \mathrm{L}$ and the displacement is given by $\mathrm{x}=\mathrm{s}$. For angles less than about $15^{\circ}$, the restoring force is directly proportional to the displacement, and the simple pendulum is a simple harmonic oscillator.

Using this equation, we can find the period of a pendulum for amplitudes less than about $15^{\circ}$. For the simple pendulum:

$$
\mathbf{T}=2 \pi \sqrt{ } \mathrm{~m} / \mathrm{k}=2 \pi \sqrt{ } \mathrm{~m} / \mathrm{mg} / \mathbf{L}
$$

Thus,

$$
\mathbf{T}=2 \pi \sqrt{ } \mathrm{~L} / \mathrm{g}
$$

For the period of a simple pendulum. This result is interesting because of its simplicity. The only things that affect the period of a simple pendulum are its length and the acceleration due to gravity. The period is completely independent of other factors, such as mass. As with simple harmonic oscillators, the period $\mathbf{T}$ for a pendulum is nearly independent of amplitude, especially if $\boldsymbol{\theta}$ is less than about $15^{\circ}$. Even simple pendulum clocks can be finely adjusted and accurate. Note the dependence of $\mathbf{T}$ on $\mathbf{g}$. If the length of a pendulum is precisely known, it can actually be used to measure the acceleration due to gravity.

### 6.6 Uniform Circular Motion and Simple Harmonic Motion

There is an easy way to produce simple harmonic motion by using uniform circular motion. Figure 6.6 shows one way of using this method. A ball is attached to a uniformly rotating vertical turntable, and its shadow is projected on the floor as shown. The shadow undergoes simple harmonic motion. Hooke's law usually describes uniform circular motions ( $\boldsymbol{\omega}$ constant)

## Chapter VI: Oscillatory Systems and Waves

rather than systems that have large visible displacements. So observing the projection of uniform circular motion, as in Figure 6.7, is often easier than observing a precise large-scale simple harmonic oscillator.

If studied in sufficient depth, simple harmonic motion produced in this manner can give considerable insight into many aspects of oscillations and waves and is very useful mathematically. In our brief treatment, we shall indicate some of the major features of this relationship and how they might be useful.


Figure 19: Example of uniform circular motion


Figure 20: Simple harmonic motion

## Chapter VI: Oscillatory Systems and Waves

### 6.7 Damped Harmonic Motion

A guitar string stops oscillating a few seconds after being plucked. To keep a child happy on a swing, you must keep pushing. Although we can often make friction and other non-conservative forces negligibly small, completely undamped motion is rare. In fact, we may even want to damp oscillations, such as with car shock absorbers.

For a system that has a small amount of damping, the period and frequency are nearly the same as for simple harmonic motion, but the amplitude gradually decreases as shown in Figure 6.8. This occurs because the non-conservative damping force removes energy from the system, usually in the form of thermal energy. In general, energy removal by non-conservative forces is described as:

$$
\mathbf{W}_{\mathrm{nc}}=\Delta(\mathrm{KE}+\mathrm{PE}),
$$

Where $\mathbf{W}_{\mathbf{n c}}$ is work done by a non-conservative force (here the damping force). For a damped harmonic oscillator, $\mathbf{W}_{\mathbf{n c}}$ is negative because it removes mechanical energy ( $\mathbf{K E}+\mathbf{P E}$ ) from the system.


Figure 21: a harmonic oscillator with a small amount of damping

### 6.8 Forced Oscillations and Resonance

Forced oscillations occur when an oscillating system is driven by a periodic force that is external to the oscillating system. In such a case, the oscillator is compelled to move at the frequency $\mathbf{v D}_{\mathbf{D}}=\boldsymbol{\omega}_{\mathbf{D}} / \mathbf{2} \boldsymbol{\pi}$ of the driving force. The physically interesting aspect of a forced oscillator is its response-how much it moves-to the imposed driving force. Let us, therefore, examine qualitatively the response of an oscillator to a driving force.

### 6.9 Waves

What do we mean when we say something is a wave? The most intuitive and easiest wave to imagine is the familiar water wave. More precisely, a wave is a disturbance that propagates, or moves from the place it was created. For water waves, the disturbance is in the surface of the water, perhaps created by a rock thrown into a pond or by a swimmer splashing the surface repeatedly. For sound waves, the disturbance is a change in air pressure, perhaps created by the oscillating cone inside a speaker. For earthquakes, there are several types of disturbances,

## Chapter VI: Oscillatory Systems and Waves

including disturbance of Earth's surface and pressure disturbances under the surface. Even radio waves are most easily understood using an analogy with water waves.

Visualizing water waves is useful because there is more to it than just a mental image. Water waves exhibit characteristics common to all waves, such as amplitude, period, frequency and energy. All wave characteristics can be described by a small set of underlying principles.

A wave is a disturbance that propagates, or moves from the place it was created. The simplest waves repeat themselves for several cycles and are associated with simple harmonic motion. Let us start by considering the simplified water wave in Figure 6.9. The wave is an up and down disturbance of the water surface. It causes a sea gull to move up and down in simple harmonic motion as the wave crests and troughs (peaks and valleys) pass under the bird. The time for one complete up and down motion is the wave's period $\mathbf{T}$. The wave's frequency is $f=1 / \mathbf{T}$, as usual.

The wave itself moves to the right in the figure. This movement of the wave is actually the disturbance moving to the right, not the water itself (or the bird would move to the right). We define wave velocity $\mathbf{v}_{w}$ to be the speed at which the disturbance moves. Wave velocity is sometimes also called the propagation velocity or propagation speed, because the disturbance propagates from one location to another.


Figure 22: an example of an idealized ocean wave

The water wave in the figure also has a length associated with it, called its wavelength $\lambda$, the distance between adjacent identical parts of a wave. ( $\lambda$ is the distance parallel to the direction of propagation.) The speed of propagation $\mathbf{v}_{w}$ is the distance the wave travels in a given time, which is one wavelength in the time of one period. In equation form, that is

## Chapter VI: Oscillatory Systems and Waves

$$
\mathbf{v}_{w}=\lambda / \mathbf{T}
$$

or

$$
\mathbf{v}_{w}=\mathbf{f} \lambda .
$$

This fundamental relationship holds for all types of waves. For water waves, $\mathbf{v}_{\boldsymbol{w}}$ is the speed of a surface wave; for sound, $\mathbf{v}_{w}$ is the speed of sound; and for visible light, $\mathbf{v}_{w}$ is the speed of light, for example.

### 6.9.1 Transverse and Longitudinal Waves

A simple wave consists of a periodic disturbance that propagates from one place to another. The wave in Figure 6.10 propagates in the horizontal direction while the surface is disturbed in the vertical direction. Such a wave is called a transverse wave or shear wave; in such a wave, the disturbance is perpendicular to the direction of propagation. In contrast, in a longitudinal wave or compressional wave, the disturbance is parallel to the direction of propagation. Figure 6.11 shows an example of a longitudinal wave. The size of the disturbance is its amplitude $\mathbf{X}$ and is completely independent of the speed of propagation $\mathbf{v}_{\boldsymbol{w}}$.


Figure 23: An example of a transverse wave


Figure 24: An example of a longitudinal wave

Waves may be transverse, longitudinal, or a combination of the two. Water waves are actually a combination of transverse and longitudinal. Sound waves in air and water are longitudinal. Their disturbances are periodic variations in pressure that are transmitted in fluids. Fluids do

## Chapter VI: Oscillatory Systems and Waves

not have appreciable shear strength, and thus the sound waves in them must be longitudinal or compressional. Sound in solids can be both longitudinal and transverse.


Figure 25: Transverse and longitudinal waves: The wave on a guitar string is transverse. The sound wave rattles a sheet of paper in a direction that shows the sound wave is longitudinal

## Check Your Understanding

Task1: label the following:


Task2: answer the following questions

1. While moving from deep water to shallow water
A. frequency of water waves decrease
B. frequency of water waves increase
C. frequency of water waves stays same

## Chapter VI: Oscillatory Systems and Waves

D. None of above
2. Waves that travel in a direction perpendicular to direction of vibration are known as
A. Transverse waves
B. Longitudinal waves
C. Sound waves
D. None of above
3. If wavelength of a wave is denoted as ' $\lambda$ ' and amplitude is denoted as ' A ', then shortest horizontal distance between any crest and trough is equal to
A. $1 / 2$ of $\lambda$
B. $\lambda$
C. $1 / 2$ of A
D. A
4. Speed of wave ' v ' is given by
A. wavelength of wave/frequency of wave
B. wavelength of wave $\times$ frequency of wave
C. frequency of wave/wavelength of wave
D. None of above
5. Which of following is not a longitudinal wave?
A. Ultrasonic wave
B. Infrasonic wave
C. Infrared wave
D. Seismic wave
6. If wavelength of a wave moving on a slinky spring with a frequency of 5 Hz is equal to 0.5 m then speed of wave is equal to
A. $0.1 \mathrm{~m} \mathrm{~s}^{-1}$
B. $2.5 \mathrm{~m} \mathrm{~s}^{-1}$
C. $10 \mathrm{~m} \mathrm{~s}^{-1}$
D. None of above
7. For a constant frequency, wavelength of an electromagnetic wave is
A. directly proportional to its velocity
B. inversely proportional to its velocity
C. independent of its velocity
D. None of above
8. All waves can be classified into two categories which are
A. Sound waves and electromagnetic waves
B. Transverse waves and electromagnetic waves
C. Longitudinal waves and electromagnetic waves
D. Transverse waves and longitudinal waves
9. If amplitude of a wave is denoted as ' A ', then vertical displacement between a crest and a trough of a wave in terms of 'A' would be
A. $1 / 2$ of A
B. A
C. 2 A
D. None of above
10. Statement related to waves that is incorrect is
A. It provides a mechanism for transfer of energy from one point to another without transfer of material
B. All waves have same speed i.e. equal to $3 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}$
C. The source of any wave is vibration or oscillation
D. All of above
11. Wave that does not belong to EM spectrum is
A. Gamma rays
B. Radio waves
C. Sound waves
D. Infrared waves

## Chapter VI: Oscillatory Systems and Waves

12. Time taken to complete a wave is termed as
A. span
B. period
C. life
D. duration
13. Any two shortest points in a wave that are in phase are termed as
A. wave distance
B. wavelength
C. phase length
D. amplitude
14. Motion that is repeated at regular intervals is termed as
A. Vibration
B. Oscillation
C. Ventilation
D. Periodic motion
15. A pendulum bob is a good example of
A. Vibration
B. Oscillation
C. Ventilation
D. Periodic motion
16. If we increase wavelength, frequency would
A. increase
B. decrease
C. remain same
D. may increase or decrease
17. Waves transfer energy from one point to other.
A. It's true
B. Its false
C. its neutral
D. None of others
18. Ups and downs in transverse waves are termed as
A. compression and rarefaction
B. crests and rarefactions
C. compressions and troughs
D. crests and troughs
19. Maximum displacement from equilibrium position is
A. frequency
B. amplitude
C. wavelength
D. period
20. Displacement-time graph depicting an oscillatory motion is
A. cos curve
B. sine curve
C. tangent curve
D. straight line
21. In s.h.m, velocity at equilibrium position is
A. minimum
B. constant
C. maximum
D. zero
22. Natural frequency of a guitar string can be changed by changing it's
A. area
B. diameter
C. length
D. stiffness
23. Over-damping results in
A. slower return to equilibrium
B. faster return to equilibrium
C. equilibrium is never achieved
D. arrhythmic return to equilibrium
24. Our eyes detect oscillations up to
A. 8 Hz
B. 9 Hz
C. 6 Hz
D. 5 Hz

## Chapter VI: Oscillatory Systems and Waves

A. displacement from equilibrium position
B. magnitude of restoring force
C. both A and B
D. force exerted on it
B. acceleration
C. time
D. frequency
26. A force that acts to return mass to it's equilibrium position is called
A. frictional force
B. restoring force
C. normal force
D. contact force
27. In cars, springs are damped by
A. shock absorbers
B. engine
C. tyres
D. brake pedals
28. If time period of an oscillation is 0.40 s , then it's frequency is
A. 2 Hz
B. 2.5 Hz
C. 3 Hz
D. 3.5 Hz
29. As amplitude of resonant vibrations decreases, degree of damping
A. increases
B. remains same
C. decreases
D. varies
30. Oscillations become damped due to
A. normal force
B. friction
C. tangential force
D. parallel force
31. In s.h.m, object's acceleration depends upon

## Chapter VI: Oscillatory Systems and Waves

38. Potential energy of system is maximum at
A. extreme position
B. mean position
C. in between extreme and mean position
D. moderate position
39. In s.h.m, acceleration is always
directed towards the
A. equilibrium position
B. mean position
C. tangent to the motion
D. downwards
40. Number of oscillations per unit time is
A. amplitude
B. wavelength
C. frequency
D. peri

## Glossary

Amplitude: the maximum displacement from the equilibrium position of an object oscillating around the equilibrium position
Antinode: the location of maximum amplitude in standing waves
Beat frequency: the frequency of the amplitude fluctuations of a wave
Constructive interference: when two waves arrive at the same point exactly in phase; that is, the crests of the two waves are precisely aligned, as are the troughs

Critical damping: the condition in which the damping of an oscillator causes it to return as quickly as possible to its equilibrium position without oscillating back and forth about this position

Deformation: displacement from equilibrium
Destructive interference: when two identical waves arrive at the same point exactly out of phase; that is, precisely aligned crest to trough

Elastic potential energy: potential energy stored as a result of deformation of an elastic object, such as the stretching of a spring

Force constant: a constant related to the rigidity of a system: the larger the force constant, the more rigid the system; the force constant is represented by $\boldsymbol{k}$

Frequency: number of events per unit of time
Fundamental frequency: the lowest frequency of a periodic waveform
Intensity: power per unit area
Longitudinal wave: a wave in which the disturbance is parallel to the direction of propagation
Natural frequency: the frequency at which a system would oscillate if there were no driving and no damping forces

Nodes: the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave

Oscillate: moving back and forth regularly between two points
Over damping: the condition in which damping of an oscillator causes it to return to equilibrium without oscillating; oscillator moves more slowly toward equilibrium than in the critically damped system

Overtones: multiples of the fundamental frequency of a sound
Periodic motion: motion that repeats itself at regular time intervals
Period: time it takes to complete one oscillation
Resonance: the phenomenon of driving a system with a frequency equal to the system's natural frequency

Resonate: a system being driven at its natural frequency
Restoring force: force acting in opposition to the force caused by a deformation
Simple Harmonic Motion: the oscillatory motion in a system where the net force can be described by Hooke's law

Simple Harmonic Oscillator: device that implements Hooke's law, such as a mass that is attached to a spring, with the other end of the spring being connected to a rigid support such as a wall

Simple pendulum: an object with a small mass suspended from a light wire or string
Superposition: the phenomenon that occurs when two or more waves arrive at the same point Transverse wave: a wave in which the disturbance is perpendicular to the direction of propagation

Under damping: the condition in which damping of an oscillator causes it to return to equilibrium with the amplitude gradually decreasing to zero; system returns to equilibrium faster but overshoots and crosses the equilibrium position one or more times

Wave velocity: the speed at which the disturbance moves. Also called the propagation velocity or propagation speed

Wavelength: the distance between adjacent identical parts of a wave
Wave: a disturbance that moves from its source and carries energy

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[^0]:    Table 2: Derived units

[^1]:    3 Two dimension Kinematics
    3.1 Vector Addition and Subtraction: Graphical Methods
    3.2 Vector Addition and Subtraction: Analytical Methods
    3.3 Projectile Motion

    Check Your Understanding

