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ENTROPY GENERATION AND NATURAL CONVECTION IN SQUARE CAVITIES WITH WAVY WALLS

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Abstract: The aim of the present work is to study the entropy generation in the natural convection process in square cavities with hot wavy walls through numerical simulations for different undulations and Rayleigh numbers, while keeping the Prandtl number constant. The results show that the hot wall geometry affects notably the heat transfer rate in the cavity. It has been found in the present numerical study that the mean Nusselt number in the case of heat transfer in a cavity with wavy walls is lower, as compared to heat transfer in a cavity without undulations. Based on the obtained dimensionless velocity and temperature values, the distributions of the local entropy generation due to heat transfer and fluid friction, the local Bejan number, and the local entropy generation are determined and plotted for different undulations and Rayleigh numbers. The study is performed for Rayleigh numbers $10^3 < \mathrm{Ra} < 10^5$, irreversibility coefficients $10^{-4} < \varphi < 10^{-2}$, and Prandtl numbers $\mathrm{Pr} = 0.71$. The total entropy generation is found to increase with increasing undulation number.

Keywords: natural convection, undulated cavity, entropy generation, Bejan number.

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INTRODUCTION

Heat transfer due to natural convection has recently been an important topic due to its wide applications in energy systems, electronic cooling devices, thermal-related plants, heating and cooling of buildings, etc. The optimal design criteria for thermal systems by minimizing their entropy generation have been a subject of great interest in these fields. Therefore, the entropy generation is employed as a key parameter for evaluating the quality in engineering applications. The second law of thermodynamics has been applied to cavity problems to determine the entropy generation due to heat and flow transport in the cavity and, consequently, to minimize the entropy generation. This new trend is important if the heat transfer community is to contribute to a good engineering solution to energy problems. The entropy generation is associated with thermodynamic irreversibility, which is present in all types of heat transfer processes.

Many works on the entropy generation with flow and heat transfer can be found in the literature. Bejan [1, 2] focused on various reasons behind the entropy generation in applied thermal engineering. The generation of entropy decreases the service life of such systems. Therefore, it makes good engineering sense to focus on irreversibility of heat transfer and fluid flow processes and try to understand the entropy generation mechanisms. Magherbi et al. [3] solved numerically the transient state of the entropy generation for laminar natural convection in a square cavity with heated vertical walls. The entropy generation and natural convection in a square cavity with differential top and bottom wall temperatures were studied by Yilbas et al. [4]. For their cavity, the total entropy generation increases

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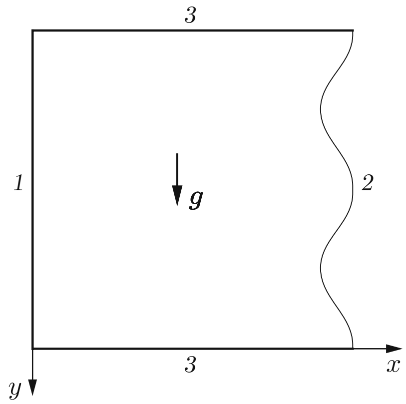

Fig. 1. Geometry of the cavity: (*1*) cold wall; (*2*) hot wall; (*3*) insulated walls.

with increasing wall temperature, which means it becomes optimum for a certain Rayleigh number. Baytas et al. [5] analyzed minimization of the entropy generation in an inclined square enclosure. Erbay et al. [6] performed an analysis to obtain the entropy generation in a partially or completely heated square cavity for the transient regime. They found that the irreversibilities were dominant due to heat transfer, whereas fluid friction irreversibilities were found to be negligible, as it was expected for natural convection. Dagtekin et al. [7] studied the prediction of the entropy generation due to natural convection in a Γ-shaped enclosure. They found that the main entropy generation occurs due to heat transfer for $\mathrm{Ra} < 10^5$. Ilis et al. [8] investigated the effect of the aspect ratio on the entropy generation in a rectangular cavity with differential heated vertical walls. They found that the total entropy generation due to fluid friction and the total entropy generation increase with increasing aspect ratio. Andreozzi et al. [9] studied the entropy generation due to natural convection in a symmetrically and uniformly heated vertical channel; one of the basic results was the increase in the global entropy generation rate as both the aspect ratio and the Rayleigh number increased.

Beside regular geometries, such as a square or a rectangle, many studies on wavy-walled enclosures were reported in the literature due to their applications in many engineering problems related to geometrical design requirements. Mahmud and Sadrul Islam [10] examined the heat transfer and laminar fluid flow characteristics inside a cavity made of two horizontal straight walls and two vertical wavy walls. In their cases, the wavy walls were assumed to have a cosine-type profile. Then, this geometry was extended to investigate heat transport through heat line visualization by Mahmud and Fraser [11].

The aim of the present work is to study the entropy generation in natural convection processes in a cavity similar to that already studied by Adjlout et al. [12] and Sabeur-Bendehina et al. [13]. The contribution of this work is the analysis of the variation of the entropy generation as a function of the Rayleigh number, undulation number, and irreversibility coefficient.

1. MATHEMATICAL MODEL

A problem of two-dimensional heat transfer in a square cavity is considered. The hot wall is wavy and has a constant temperature T_h. The cold wall is opposite to the latter and has a constant temperature T_c, while the other sides are insulated. The Rayleigh number is varied up to $\mathrm{Ra} = 10^5$, while the Prandtl number is fixed to be $\mathrm{Pr} = 0.71$. Figure 1 shows the geometry of the cavity under consideration and the coordinate system.

The thermophysical properties of the fluid in the flow model are assumed to be constant, except for density variations responsible for the body force term in the momentum equation. The Boussinesq approximation is invoked for the fluid properties to relate density changes to temperature changes and to couple in this way the temperature field to the flow field. The governing equations for the steady natural convection flow includes the equations of conservation of mass, momentum, and energy:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,$$

$$u\,\frac{\partial u}{\partial x}+v\,\frac{\partial u}{\partial y}=-\frac{1}{\rho}\,\frac{\partial p}{\partial x}+\nu\Big(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\Big),$$

$$u\,\frac{\partial v}{\partial x}+v\,\frac{\partial v}{\partial y}=-\frac{1}{\rho}\,\frac{\partial p}{\partial y}+\nu\Big(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\Big)+g\beta(T-T_c), \tag{1}$$

$$u\,\frac{\partial T}{\partial x}+v\,\frac{\partial T}{\partial y}=\alpha\Big(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\Big).$$

Here x and y are the Cartesian coordinates, u and v are the velocity vector components in the x and y directions, respectively, ρ is the density, ν is the kinematic viscosity, p is the pressure, α is the thermal diffusivity, β is the thermal expansion coefficient, and T is the temperature.

In the dimensionless variables

$$X=\frac{x}{L},\quad Y=\frac{y}{L},\quad U=\frac{uL}{\alpha},\quad V=\frac{vL}{\alpha},\quad \theta=\frac{T-T_c}{T_h-T_c},$$

$$P=\frac{pL^2}{\rho\alpha^2},\quad \mathrm{Pr}=\frac{\nu}{\alpha},\quad \mathrm{Ra}=\frac{g\beta(T_h-T_c)L^3\mathrm{Pr}}{\nu^2}$$

(L is the characteristic length), the governing equations (1) reduce to

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0,$$

$$U\,\frac{\partial U}{\partial X}+V\,\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\mathrm{Pr}\Big(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\Big),$$

$$U\,\frac{\partial V}{\partial X}+V\,\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\mathrm{Pr}\Big(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\Big)+\mathrm{Ra}\,\mathrm{Pr}\,\theta, \tag{2}$$

$$U\,\frac{\partial \theta}{\partial X}+V\,\frac{\partial \theta}{\partial Y}=\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}.$$

The boundary conditions for the hydrodynamic problem are impermeability and non-slipping on all cavity walls. For the thermal problem, the horizontal walls are kept adiabatic, while the vertical walls are kept at a constant temperature. The hot wall is kept with $\theta_h=0.5$, and the cold wall is kept with $\theta_c=-0.5$.

In the entire domain, zero initial conditions are imposed for the velocity field. The shape of the wavy vertical wall is taken to be sinusoidal:

$$f(y)=1-A(1-\cos(2\pi ny))$$

(n and A are the number of undulations and the amplitude, respectively). The amplitude is fixed in this study at 0.05.

The heat transfer rate due to convection in an enclosure is obtained from the Nusselt number calculation. On the wavy wall, the local and mean Nusselt numbers are expressed as follows:

$$\mathrm{Nu}_l=\frac{\partial\theta}{\partial n},\qquad \overline{\mathrm{Nu}}=\frac{1}{s}\int_0^s\frac{\partial\theta}{\partial n}\,ds.$$

The dimensionless local entropy generation due to heat transfer and the fluid friction (S_{LHT} and S_{LFF}, respectively) for a two-dimensional heat and fluid flow in the Cartesian coordinates can be written as

$$S_{\mathrm{LHT}}=\Big(\frac{\partial\theta}{\partial X}\Big)^2+\Big(\frac{\partial\theta}{\partial Y}\Big)^2,$$

$$S_{\mathrm{LFF}}=\varphi\Big\{2\Big[\Big(\frac{\partial U}{\partial X}\Big)^2+\Big(\frac{\partial V}{\partial Y}\Big)^2\Big]+\Big(\frac{\partial U}{\partial Y}+\frac{\partial V}{\partial X}\Big)^2\Big\}.$$

Grid validation: comparison of results obtained on different grids

Grid	$\overline{\text{Nu}}$		
	$\text{Ra} = 10^5$	$\text{Ra} = 10^4$	$\text{Ra} = 10^3$
29×34	3.907	2.225	1.095
40×48	3.820	2.219	1.072
67×80	3.760	2.156	1.067
83×100	3.720	2.149	1.064

where φ is the parameter determining the contributions of heat transfer and friction contributions to the entropy generation:

$$\varphi = \frac{S_{\text{LFF}}}{S_{\text{LHT}}} = \frac{\mu T_0}{k}\Big(\frac{\alpha}{L\Delta T}\Big)^2,$$

μ is the dynamic viscosity, $T_0 = (T_h + T_c)/2$, and k is the thermal conductivity.

The local entropy generation S_l is the sum of S_{LHT} and S_{LHT}:

$$S_l = \Big[\Big(\frac{\partial T}{\partial X}\Big)^2 + \Big(\frac{\partial T}{\partial Y}\Big)^2\Big] + \varphi\Big\{2\Big[\Big(\frac{\partial U}{\partial X}\Big)^2 + \Big(\frac{\partial V}{\partial Y}\Big)^2\Big] + \Big(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\Big)^2\Big\}. \quad (3)$$

The total entropy generation due to heat transfer and fluid friction (S_{THT} and S_{TFF}, respectively) are obtained by integrating the local entropy generation over the system volume:

$$S_{\text{THT}} = \int\limits_V S_{\text{LHT}}\, dV, \qquad S_{\text{TFF}} = \int\limits_V S_{\text{LFF}}\, dV, \qquad S_t = S_{\text{THT}} + S_{\text{TFF}}.$$

The local Bejan number indicates the strength of the entropy generation due to heat transfer irreversibility:

$$\text{Be} = \frac{S_{\text{LHT}}}{S_l}.$$

At $\text{Be} \gg 1/2$, the irreversibility due to heat transfer dominates; at $\text{Be} \ll 1/2$, the irreversibility due to viscous effects dominates; finally, at $\text{Be} = 1/2$, the heat transfer and fluid friction irreversibilities are equal. The heat transfer irreversibility is the only origin of the entropy generation at $\text{Be} = 1$; at $\text{Be} = 0$, the fluid friction irreversibility is the only origin of the entropy generation.

2. GRID VALIDATION

In order to study the precision of computations, various grids were tested. The table shows the results of computations performed on various grids for three Rayleigh numbers and for a wavy wall at $n = 3$. The mean Nusselt number, which describes heat transfer, can be considered as a good criterion for validation of our computations. The choice of the 67×80 grid used in the rest of the study is justified by the fact that the results obtained on this grid differ from the results obtained on a finer grid by less than 4%.

3. NUMERICAL PROCEDURE

In the dimensionless coordinates, the governing equations are solved by a finite-volume method with a pressure-correction procedure introduced by Patankar [14]. Detailed information about the numerical procedure and convergence criteria can be found in [13]. From the known temperature and velocity fields given by solving Eqs. (2), the local entropy generation for the entire cavity is easily obtained by Eq. (3) after knowing the temperature gradient at the walls of the cavity.

4. RESULTS AND DISCUSSION

In this paragraph, we discuss the results obtained for the cavity with four undulations ($n = 4$) at $\varphi = 0.0001$ and different Rayleigh numbers.

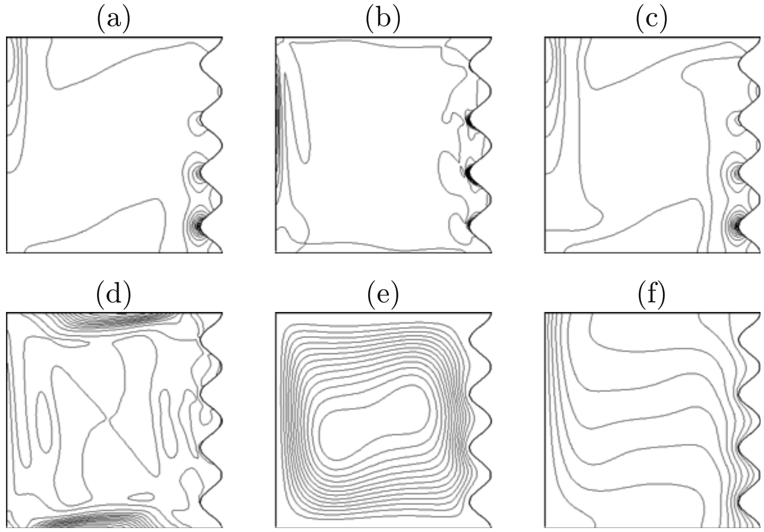

Fig. 2. Calculated results for a cavity with four undulations ($n = 4$) at $\varphi = 0.0001$ and $\mathrm{Ra} = 10^5$: (a) local entropy generation due to heat transfer; (b) local entropy generation due to fluid friction; (c) local entropy generation due to heat transfer and fluid friction; (d) Bejan number contours; (e) streamlines; (f) isotherms.

Figure 2 shows the results calculated at $\mathrm{Ra} = 10^5$. It is clear that the undulated wall affects the geometrical shape acquired by the cell, as it is noticed on the streamline patterns. In particular, near the hot wall, the streamlines converge toward the wall after each crest and diverge after each trough. This behavior has a repercussion on heat transfer by convection near the wall. Indeed, when the normal velocity component makes the streamline approach the wall, the heat transfer intensity increases. It should be noted that acceleration of the thermo-convective flow leads to a diagonally stretched cell. Near the crest, the streamlines are near the wall just after the crest. The isotherm distribution shows the same feature near the latter region (see Fig. 2). This observation proves the diminution of the thermal boundary layer thickness just after the crest. The flow stays stratified in the core region of the cavity. The boundary layer thickness alternatively increases and then decreases along the hot wall. In particular, near the hot wall, the streamlines converge toward the wall after each crest and diverge after each trough. The isotherms in the cavity are not parallel to the vertical walls due to geometry waviness and a strong fluid flow. The local entropy generation due to fluid friction is considerably greater than the local entropy generation due to heat transfer. Hence, the local entropy generation map is quite similar to the heat transfer map. Figures 2–4 show that the entropy generation is mainly located at the lower corner of the heated wavy wall and at the upper corner of the cooled wall; this is due to the thermal velocity gradients in the above-mentioned regions (see Figs. 2e, 2f, 3e, 3f, 4e, and 4f).

For high Rayleigh numbers, the entropy generation isolines are located on the active sides (heated and cooled walls); this is also due to considerable thermal velocity gradients, as described by the isotherms and streamlines.

The domination of heat transfer irreversibilities in all cases studied can be seen in Figs. 3 and 4, which depict the distributions of the local Bejan number in the cavity. The obtained results show that the local Bejan number varies from 0 to 1. It is also noticed that the local Bejan number increases, again showing that the thermal irreversibility dominates in all cases and all undulations of the cavity. At low Rayleigh numbers, the fluid is almost motionless in the cavity, and the irreversibility is entirely dominated by heat transfer. The Bejan contours show symmetric distributions, and the heat transfer irreversibility occurs near the wavy walls as well as in the core region of the cavity. The convection flow becomes stronger at high Rayleigh numbers, resulting in distortion of the isotherms. As the Rayleigh number increases, the zone of irreversibility in the core region becomes narrow.

Figure 5 illustrates the total entropy generation as a function of the Rayleigh number for all cavities tested at $\varphi = 0.0001$. The distribution patterns of the entropy generation are very much similar for all undulations and

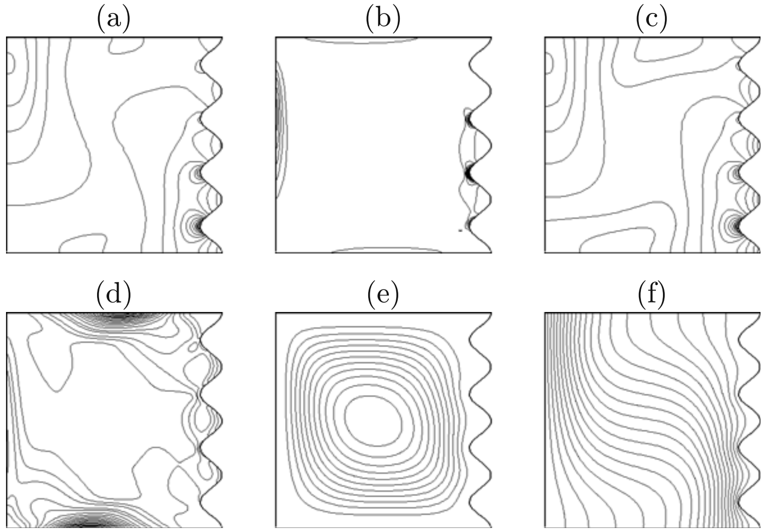

Fig. 3. Calculated results for a cavity with four undulations ($n = 4$) at $\varphi = 0.0001$ and $\mathrm{Ra} = 10^4$ (notation the same as in Fig. 2).

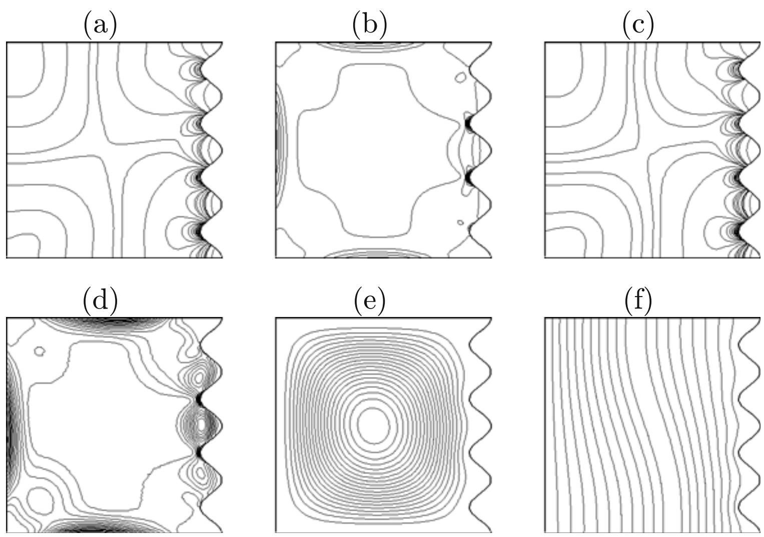

Fig. 4. Calculated results for a cavity with four undulations ($n = 4$) at $\varphi = 0.0001$ and $\mathrm{Ra} = 10^3$ (notation the same as in Fig. 2).

for the square cavity. It is clearly seen that an increase in the Rayleigh number induces a linear increase in the entropy generation value. The total entropy generation also increases as the undulation number increases.

Figure 6a shows the variation of the total entropy generation with the undulations number for $\mathrm{Ra} = 10^3$ and different values of φ. It is clearly seen that the variation of the total entropy generation is similar for two values ($\varphi = 0.0001$ and $\varphi = 0.01$) for all undulations, but the total entropy generation for $\varphi = 0.001$ is much higher than in the other cases studied. However, for the other values of the Rayleigh number, the total entropy generation increases with increasing undulations number, but it is smaller for $\varphi = 0.001$ and $\varphi = 0.0001$ than for $\varphi = 0.01$ and reaches a maximum value for the cavity with $n = 4$ undulations. This feature is particularly well seen at $\mathrm{Ra} = 10^4$ (cf. Figs. 6b and 6c).

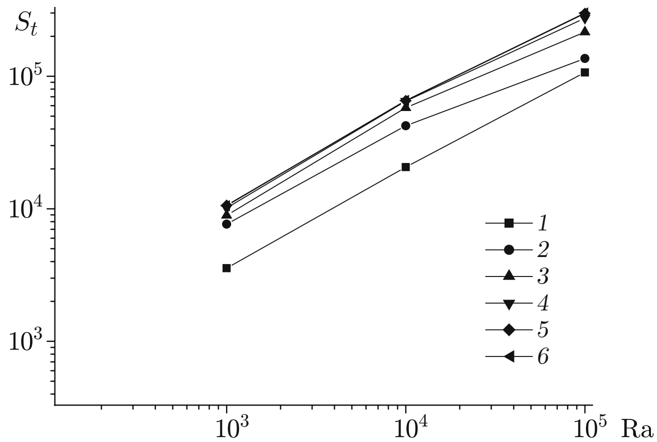

Fig. 5. Entropy generation versus the Rayleigh number at $\varphi = 0.0001$: square cavity (*1*); cavity with undulations $n = 2$ (*2*), 3 (*3*), 4 (*4*), 5 (*5*), and 6 (*6*).

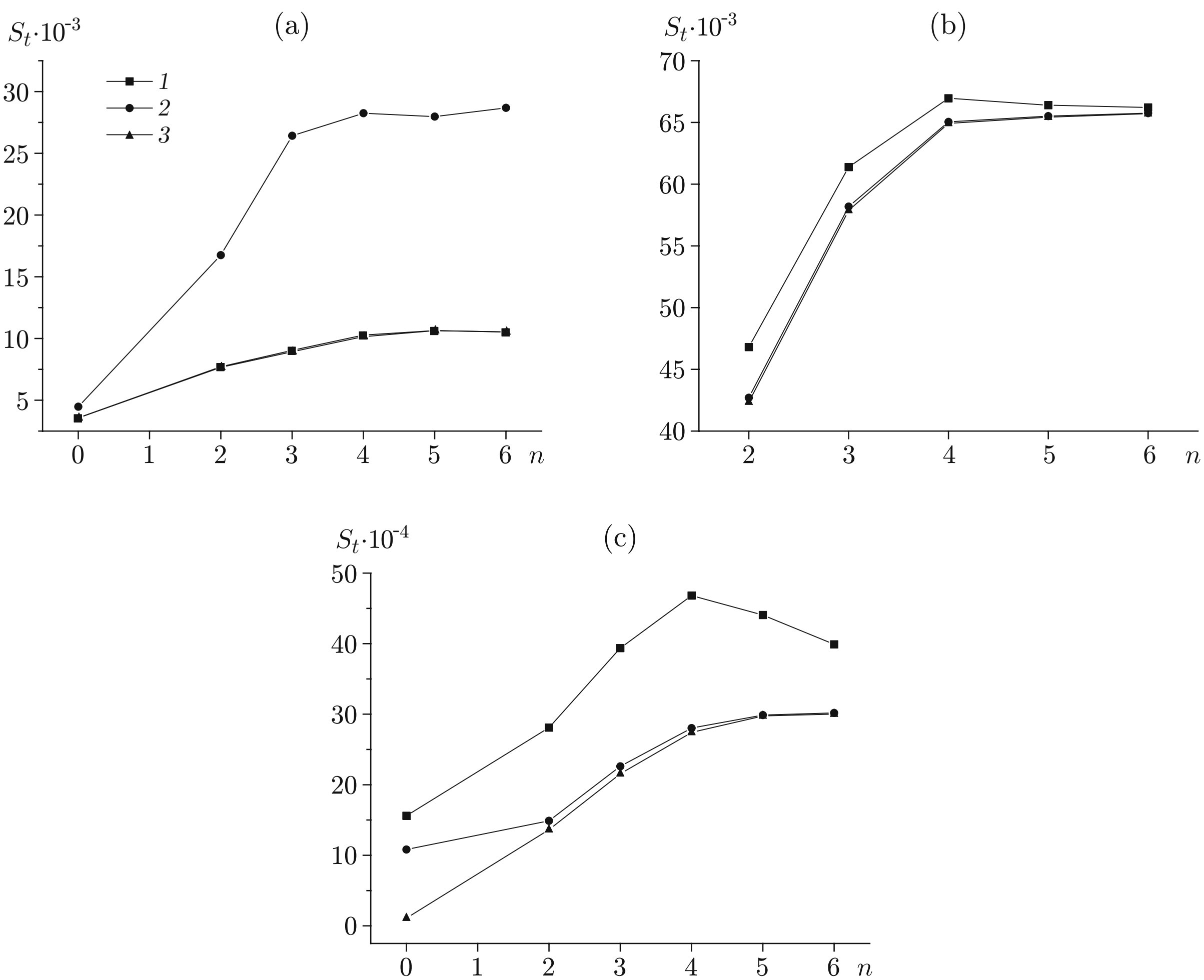

Fig. 6. Total entropy generation versus the undulation number for Ra = 10^3 (a), 10^4 (b), and 10^5 (c): $\varphi = 0.01$ (*1*), 0.001 (*2*), and 0.0001 (*3*).

CONCLUSIONS

For an undulated cavity with different undulations, the variation of the local and total heat transfer and fluid friction irreversibilities and entropy generation were investigated. The mean Bejan number was used to evaluate the total entropy generation due to heat transfer in the entire domain. The heat transfer and fluid friction irreversibilities vary considerably with the number of undulations. The highest entropy generation in the cavity is observed at $\mathrm{Ra} = 10^3$ and $\varphi = 0.001$ for all undulation numbers.

The heat transfer is strongly dominant, and the total entropy generation increases with an increase in the number of undulations. However, at high Rayleigh number values ($\mathrm{Ra} = 10^5$) and all values of φ, a peak point for the maximum total entropy generation exists at $n = 4$, and then its value decreases.

In this study, it was possible to observe that the thermal and hydrodynamic problems are highly coupled. For thermophysical configurations involving natural convection, the geometry with undulations is a better option. It could be applied for solar thermal collectors or electronic circuits found in many energy systems.

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